Energy Engineering

Analysis of heat transfer and fluid flow in thermal equipment

MASTER THESIS

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Summary

Heat transfer and flow fluid in thermal equipment is a concerned and basic topic in thermal area. The main objective of this thesis is to cultivate basic skills of analyzing a thermal problem and selecting an appropriate method to solve it according to the background.

In the following context, basic modes of heat transfer and classification of heat exchangers will be presented and Navier-Stoke equation for incompressible fluids will be introduced.

In the next chapter, a problem of composed wall will be solved using analytical and numerical method. Before solving the problem, discretization equation for one-dimension steady conduction is deduced.

Later, convection-diffusion equation is developed and a new convection problem of an isotherm tube is studied. In this case, step-by-step method is used to obtain final results.

Last chapter is about heat exchangers. Methodologies to solve design and predict problem of heat exchangers are introduced: F-factor method and e-NTU method. The next step is to apply them into some problem: a simple co-current flow HX and a simple counter flow HX. Both analytical and numerical method are used to verify each other. Finally, an advanced double tube is studied.

Keywords *– heat transfer, conduction, convection-diffusion equation, analytical method, numerical method, step-by-step method, heat exchanger*

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1. Glossary

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1D	One-dimensional
2D	Two-dimensional
BC	Boundary conditions
CDS	Central difference scheme
CV	Control volume
EDS	Exponential-difference scheme
FSM	Fractional step method
FV	Finite volume
GS	Gauss-Seidel method
HT	Heat transfer
HX	Heat exchangers
QUICK	Quadratic upwind interpolation for convective kinematics
SUDS	Second-order upwind difference scheme
TDMA	Tri-diagonal matrix algorithm
UDS	Upwind difference scheme

2. Preface

2.1. Origin of project

The project is developed based on some courses (Thermal Equipment, Heat Exchangers and Numerical Methods in Heat Transfer) taken by Professor Carlos David Pérez Segarra as well as a research work (Numerical heat transfer and fluid flow) under the guidance of him. This project will focus more advanced studies on the analysis of heat transfer and fluid flow in a double tube heat exchanger using semi-analytical and numerical methods.

In this project, there will be mainly three parts: heat conduction, heat convection and heat exchangers and in each part one case will be studied as well as some parametric studies.

In the part of heat conduction, firstly heat conduction discretization of two-dimension is introduced. Then a one-dimensional solid composed wall problem will be solved by both methods and following by parametric study of heat source and geometry conditions of the solid wall.

In convection part, convection-diffusion equation comes first and the next is to solve a problem of isothermal tube using equation mentioned before. The problem will also be settled by analytical and numerical methods and three fluids will be studied separately and compared together. At the end of this chapter, parametric study is given: length and diameter of the tube are changed.

The last chapter are about double tube exchangers. Basic methodologies to solve HX problem semi-analytically are explained: F-factor and e-NTU methods. Then a simple double tube heat problem is solved with two different flows: co-current flow and counter flow. Both kinds of heat exchangers will be solved parallel in semi-analytical and numerical methods. Similarly, parametric studies are necessary: geometry condition and different types of fluids in heat exchangers.

2.2. Previous Requirements

In order to finish this project, some previous requirements are needed which are listed below:

Fundamentals of thermodynamics

Fundamentals of fluid mechanics

Fundamentals of heat transfer

Basic knowledge of computer language

3. Introduction

The energy issue is one of the most concerning issues that the human being is facing, so the studying of energy saving will be important for the sustainable development of each countries. While heat transfer is a discipline that studies process caused by temperature differences and it plays a vital role in industrial and chemical areas. With the further studying in this subject, heat transfer shows potential in many other areas.

3.1. Heat transfer

Heat Transfer is the science that seeks to predict the energy transfer that may be take place between material bodies as a result of a temperature difference. Whenever there is a temperature difference in a medium or between media, heat transfer must occur.

There are basic modes of heat transfer: conduction, convection and radiation. Heat conduction is the most common that it occurs in nature [1]. It's the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.

Fourier's law is introduced to solve the heat conduction problems. It states an empirical relationship between the conduction rete in a material and the temperature gradient in the direction of energy flow, which was proposed by Fourier in 1822 [2]. He drew a conclusion that the heat flux from heat conduction is proportional to the magnitude of temperature gradient and opposite to its direction. For a steady heat conduction process, the heat flux in x-direction could be expressed as:

$$q_x = -k\frac{dT}{dx}$$

Where, q_x is the heat flux (W/m^2) in the positive x direction and $\frac{dT}{dx}$ is the temperature gradient (K/m) in the direction of heat flow. k is the thermal conductivity of material (W/mK) and it's a constant property? Fourier's law is the basic law to solve the heat transfer problems and it could also be applied into unsteady multi-dimensional conduction problems.

Convection is the mode of energy transfer between a surface and the adjacent liquid or gas that is in motion, and it involves the combined of conduction and fluid motion [3]. There're two types of convection: natural and forced convection. Natural convection refers to the fluid motion generated by density differences of fluids not by any external source while forced convection is generated by an external source.

Newton's law of cooling was formulated by Isaac Newton in 1701 [4] that illustrates the relationship that obtained from an empirical observation of convective cooling of hot bodies. It says that the rate of heat loss of a body is directly proportional to the differences between the body and its surroundings, expressed as:

$$q = h(T_h - T_c)$$

Where, q is the rate of heat loss (W/m^2) and h is heat transfer coefficient (W/m^2K). T_h and T_c represent the temperatures of hot body and cold body respectively.

Radiation occurs when two bodies are at different temperatures and separated by distance [5]. Unlike conduction and convection, there is no media in the radiation heat transfer. It happens due to the electromagnetic waves that exist in the atmosphere.

Stefan-Boltzmann law describes the power radiated from a black body in terms of its temperature [6]. It states that the total energy radiated is directly proportional to the fourth power of the blackbody's thermodynamic temperature, which could be expressed as:

$$q_b = \sigma T^4$$

Where, q_b is the rate of radiated energy by blackbody (W/m^2) and σ is called Stefan-Boltzmann constant with a value of $5.670373 \times 10^{-8} W/m^{-2}K^{-4}$.

For a body that doesn't absorb all incident radiation emits less total energy than a blackbody, which is characterized by an emissivity ε, we have

$$q = \varepsilon\sigma T^4$$

Where ε is always smaller than 1.

3.2. Thermal equipment

Thermal equipment or called thermal systems refers to a multipart assembly of coupled components, showing a common structured behavior like heat exchangers, heat pump, refrigeration and air conditioning systems. All this equipment is used to do the work related to production, consumption or processing of thermal energy. In this report, main focus will be on heat exchangers.

A heat exchanger is a device used to transfer heat between two or more fluids, which could be single or two phases. It's mainly used in cooling and heating process. The fluids may be separated by a solid wall or may be in direct contact.

Heat exchangers can be classified by four criterions: types of fluids, interaction between currents, flow configuration and technological application. Among them, the classification by flow arrangement is the most common one. According to the flow configuration, heat exchangers can be divided into four types: counter flow, co-current flow, cross flow and hybrids such as E shell with a number of tube passes.

In a counter flow heat exchanger, there’re two fluids flow parallel to each other but go in different direction, which is shown as Figure 1. This type of heat exchanger has the highest efficiency compared with the following types because it allows the largest change in temperature of both fluids. The efficiency of a heat exchange is the rate of actual heat transferred and theoretical maximum amount of heat that can be transferred.

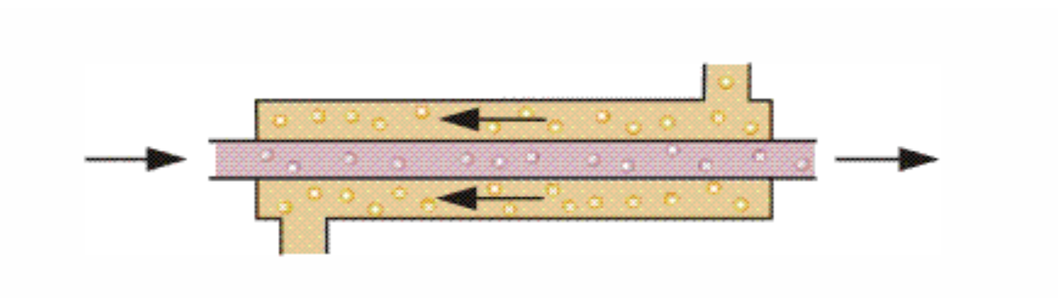

Figure 1 Counter flow [7]

Figure 2 shows a co-current flow, in which the fluid flow parallel to each other and also in same direction. This one has a lower efficiency but provides a more uniform wall temperature.

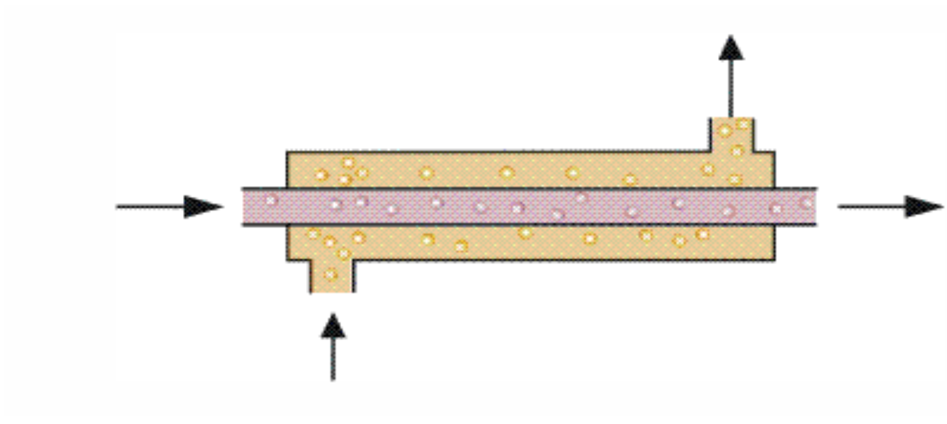

Figure 2 Co-current flow [7]

Cross flow heat exchangers are those in which the two fluids are not in direct contact and flow roughly perpendicular to each other. They’re usually used in a cooling system like condenser and ventilation system where heat is required to be transferred from one airstream to another. Figure 3 shows a cross flow heat exchanger.

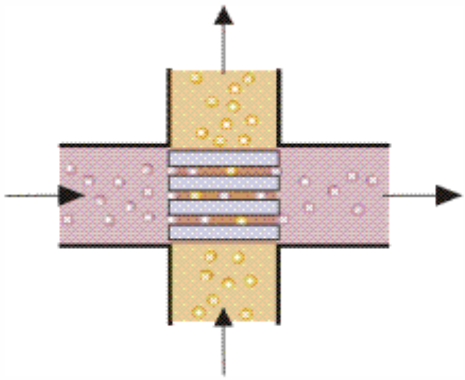

Figure 3 Cross flow [7]

As for hybrids flow heat exchanger, it’s usually a combination of the above flow type. For an example, Figure 4 shows a shell and tube exchanger.

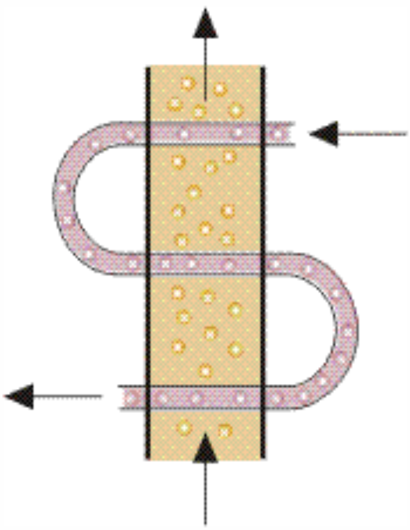

Figure 4 Hybrids flow [7]

3.3. Objectives

The main objectives of this thesis are:

Consolidate and enhance basic knowledge of thermodynamics, fluid mechanics and heat and mass transfer.

Acquire basic training in understanding types and usefulness of different types of heat exchanger.

Acquire a very solid and yet flexible (adaptation to different geometries or phenomenology) in calculation of heat exchangers by analytical and numerical methods.

Know the different levels of calculation of heat exchangers (porosity method, detailed calculation dimensional flows, solving the NS equations) and their combination.

Be able to select the most suitable thermal equipment from the energy point of view for each application (industry or services) and analyze the performance of existing equipment.

4. Conduction heat transfer problems

This chapter is dedicated to the conduction heat transfer problems. It will begin with the process of heat conduction discretization and then solve a heat conduction problem of a composed wall.

The objective of this chapter is to obtain heat conduction discretized equation from the general differential equation in an unsteady 2D case and solve a conduction problem using both numerical and analytical methods and compare results. Besides, this chapter is going to introduce some preparatory work for later chapters.

4.1. Heat conduction discretized equation

For an unsteady 2D heat conduction case, the differential equation is:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \dot{q_v}$$

Where, ρ is the density, C_p is the specific heat, λ is the thermal conductivity and q_v is the internal hear source.

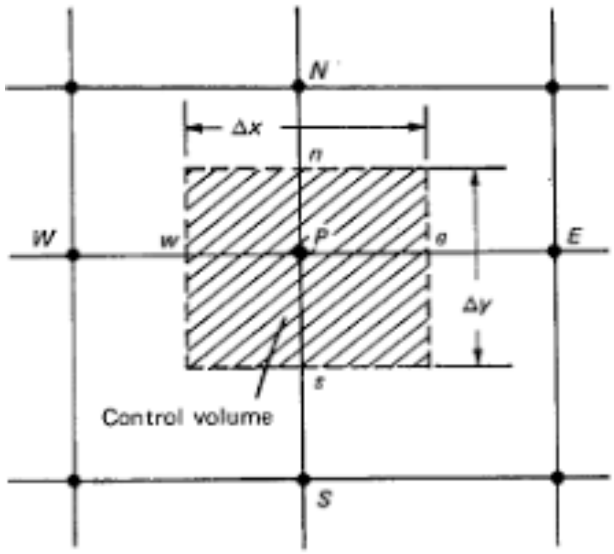

Figure 5 Control volume [8]

As Figure 5 shows, the gird point P is an internal point which is surrounded by four points: W (west), E (east) in x direction and N (north), S (south) in y direction. According to energy balance, we have the following equation for all internal nodes:

$$\beta\left(\dot{Q}_W^{n+1} + \dot{Q}_E^{n+1} + \dot{Q}_N^{n+1} + \dot{Q}_S^{n+1} + \dot{Q}_{gen}^{n+1}\right) + (1-\beta)\left(\dot{Q}_W^{n} + \dot{Q}_E^{n} + \dot{Q}_N^{n} + \dot{Q}_S^{n} + \dot{Q}_{gen}^{n}\right) = \dot{Q}_{ac}$$

There 're three finite difference methods to solve the differential equations depending the value of β: explicit method, implicit method and Crank-Nicolson method.

When β is 0, it's the explicit scheme using a forward difference. Thus, the above equation could be written as:

$$\dot{Q}_W^{n} - \dot{Q}_E^{n} + \dot{Q}_S^{n} - \dot{Q}_N^{n} + \dot{Q}_{gen}^{n} = \dot{Q}_{ac}$$

When β is 0.5, it's the Crank-Nicolson scheme using the central difference. Thus, the above equation could be written as:

$$0.5\left(\dot{Q}_W^{n+1} + \dot{Q}_E^{n+1} + \dot{Q}_N^{n+1} + \dot{Q}_S^{n+1} + \dot{Q}_{gen}^{n+1}\right) + 0.5\left(\dot{Q}_W^{n} + \dot{Q}_E^{n} + \dot{Q}_N^{n} + \dot{Q}_S^{n} + \dot{Q}_{gen}^{n}\right) = \dot{Q}_{ac}$$

When β is 0, it's the implicit scheme using a backward difference. Thus, the above equation could be written as:

$$\dot{Q}_W^{n+1} - \dot{Q}_E^{n+1} + \dot{Q}_S^{n+1} - \dot{Q}_N^{n+1} + \dot{Q}_{gen}^{n+1} = \dot{Q}_{ac}$$

Each scheme has its own advantages and discussion, either in accuracy or stability. Considering these factors, the implicit scheme is selected and super index n+1 will be neglected for convenience in writing. Therefore, the equation using implicit scheme can be simplified as:

$$\dot{Q_W} - \dot{Q}_E + \dot{Q}_S - \dot{Q}_N + \dot{Q_{gen}} = \dot{Q}_{ac}$$

Applying Fourier's law to the left side of equation, new formula will be obtained:

$$-\lambda_w \frac{T_P - T_W}{d_{PW}} S_w + \lambda_e \frac{T_E - T_P}{d_{PE}} S_e - \lambda_s \frac{T_P - T_S}{d_{PS}} S_s + \lambda_n \frac{T_N - T_P}{d_{PN}} S_n + \dot{q_v} V_p$$

Where, $T_P = T[i,j]$ temperature of principal point

$T_W = T[i-1,j]$, temperature of west node

$T_E = T[i+1,j]$, temperature of east node

$T_S = T[i,j-1]$, temperature of south node

$T_N = T[i,j+1]$, temperature of north node

As for the right side, the accumulated energy could be transferred into:

$$\dot{Q}_{ac} = \frac{\partial T}{\partial t}\int_{cv} \rho u \, dV \approx \bar{\rho}\overline{C_p}V_p \frac{T_P^{n+1} - T_P^n}{\Delta t}$$

Where, density ρ and specific heat C_P are two thermophysical properties depending on the temperature at each time constant and basically average values are used from time n to time n+1.

$$\bar{\rho} = \frac{\rho[T_P^{n+1}] + \rho[T_P^n]}{2}$$

$$\bar{C_p} = \frac{C_P[T_P^{n+1}] + C_p[T_P^n]}{2}$$

Above all, the heat conduction discretized equation for 2D case could be obtained:

$$a_P T_P = a_W T_W + a_E T_E + a_S T_S + a_N T_N + b_P$$

The discretization coefficients for internal nodes are:

$$a_W = \frac{\lambda_w}{d_{PW}} S_w$$

$$a_E = \frac{\lambda_e}{d_{PE}} S_e$$

$$a_S = \frac{\lambda_s}{d_{PS}} S_s$$

$$a_N = \frac{\lambda_n}{d_{PN}} S_n$$

$$a_p = a_W + a_E + a_S + a_N + \frac{\bar{\rho}\overline{C_p}V_p}{\Delta t}$$

$$b_P = \dot{q}_v V_p + \frac{\bar{\rho}\overline{C_p}V_p T_P^n}{\Delta t}$$

When discretization equations are obtained, the next step is to seek solution to solve them. Generally, Gauss-Seidel method, TDMA and line-by-line method are most used to get answers.

4.2. Problems of composed wall

4.2.1. Problem description

A one-dimensional solid composed of two different material and surrounded by fluid. The objective is to find the temperature's distribution, $T(x)$, across the solid and, specifically, the values of the temperatures at the walls:T_{w1}, T_{w1-2}, T_{w2}. The configuration is shown below.

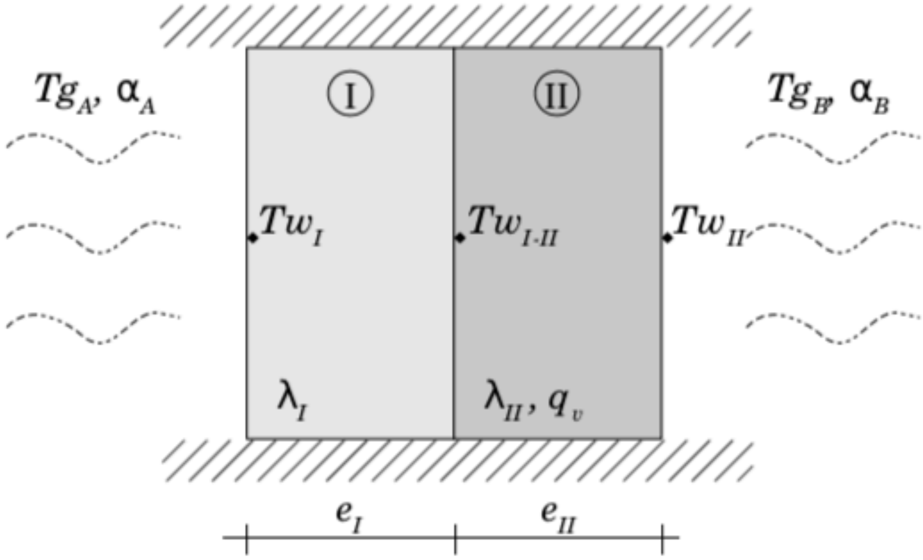

Figure 6 Composed wall [9]

Initial data and properties are shown in the table below.

Geometry condition	
$Tg_A(°C)$	40
$Tg_B(°C)$	20
$\alpha_A(W/m^2K))$	10
$\alpha_B(W/m^2K))$	1000
$e_1(m)$	0.1
$e_2(m)$	0.2
$\lambda_1(W/mK)$	0.2
$\lambda_2(W/mK)$	400
$q_{v1}(W/m^3)$	100
$q_{v2}(W/m^3)$	1000

4.2.2. Analytical method

For the plane walls, we have temperature and heat flux distribution:

$$T(x) = -\frac{q_v}{2\lambda}x^2 + C_1 x + C_2$$

$$q_x = q_v x - \lambda x$$

Therefore, for each material we have:

$$T_1(x) = -C_1 x + C_2$$

$$T_2(x) = -\frac{q_v}{2\lambda}x^2 + C_3 x + C_4$$

Then we apply boundary conditions to these equations, we'll get some new equations:

$$T_1(0) = T_{W1}$$

$$T_1(e_1) = T_{w1-2}$$

The temperature distribution in material A will be:

$$T_1(x) = \frac{T_{w1-2} - T_{W1}}{e_1}x + T_{W1}$$

$$T_2(e_1) = T_{w1-2}$$

$$T_2(e_1 + e_2) = T_{w2}$$

The temperature distribution in material B will be:

$$T_2(x) = -\frac{q_{v2}}{2\lambda}(x - e_1)(x - e_1 - e_2) + \frac{T_{W2} - T_{W1-2}}{e_2}(x - e_1) + T_{W1-2}$$

Finally, we introduce energy conservation at the boundary points, two new equations would be obtained:

$$\alpha_A(Tg_A - T_{w1}) = -\lambda_1 \frac{dT_1}{dx}_{x=0} = -\lambda_1 \frac{T_{w1-2} - T_{w1}}{e_1}$$

$$\alpha_B(T_{w2} - Tg_B) = -\lambda_2 \frac{dT_2}{dx}_{x=e_1+e_2} = \frac{q_v}{2}e_2 - \lambda_2 \frac{T_{w2} - T_{w1-2}}{e_2}$$

While, in this case we have three unknown terms, we need another equation:

$$-\lambda_1 \frac{dT_1}{dx}_{x=e_1} = -\lambda_2 \frac{dT_2}{dx}_{x=e_1}$$

$$-\lambda_1 \frac{T_{w1-2} - T_{w1}}{e_1} = -\frac{q_v}{2} e_2 - \lambda_2 \frac{T_{w2} - T_{w1-2}}{e_2}$$

4.2.3. Numerical method

In the numerical method, we use finite volume method to divide the domain in a finite numbers of control volumes which depends on required precision. The method locates a node at the centroid of each control volume and assumes that the properties of fluids are constant. The material is consisted of two parts each part could be divided into NA and NB control volumes. So, in this case, there are five types of nodes: i=1, i=2 to NA (internal nodes in material A), i=NA+1, i=NA+2 to NB (internal nodes in material B) and i=NA+NB+1.

Before solving the problem, I made some assumptions:

One-dimensional and steady state

Constant thermos-physical properties for the material

Then the discretization equations could be obtained. They're listed below:

a) $i = 1$

$$Q_{conv} - Q_E + Q_{v1} = 0$$
$$\alpha_A(Tg_A - T_P) + \lambda_1 \left(\frac{T_E - T_P}{d_{PE}}\right) = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = 0$$
$$a_E = \frac{\lambda_1}{dx_1}$$
$$a_P = \alpha_A + \frac{\lambda_1}{dx_1}$$
$$b_P = \alpha_A Tg_A$$

b) $i = 2\ to\ NA$

$$Q_W - Q_e + Q_{v1} = 0$$
$$-\lambda_1 \left(\frac{T_P - T_W}{d_{WP}}\right) + \lambda_1 \left(\frac{T_E - T_P}{d_{PE}}\right) = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = \frac{\lambda_1}{dx_1}$$
$$a_E = \frac{\lambda_1}{dx_1}$$
$$a_P = \frac{\lambda_1}{dx_1} + \frac{\lambda_1}{dx_1}$$
$$b_P = 0$$

c) $i = NA + 1$

$$Q_W - Q_e + Q_{v1} + Q_{v2} = 0$$
$$-\lambda_1 \left(\frac{T_P - T_W}{d_{WP}}\right) A + \lambda_2 \left(\frac{T_E - T_P}{d_{PE}}\right) A + q_{v2} V_2 = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = \frac{\lambda_1}{dx_1}$$
$$a_E = \frac{\lambda_2}{dx_2}$$
$$a_P = \frac{\lambda_1}{dx_1} + \frac{\lambda_2}{dx_2}$$
$$b_P = q_{v2} \frac{V_2}{A}$$

d) $i = NA + 1 \; to \; NB$

$$Q_W - Q_e + Q_{v2} = 0$$
$$-\lambda_2 \left(\frac{T_P - T_W}{d_{WP}}\right) A + \lambda_2 \left(\frac{T_E - T_P}{d_{PE}}\right) A + q_{v2} V_2 = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = \frac{\lambda_2}{dx_2}$$
$$a_E = \frac{\lambda_2}{dx_2}$$
$$a_P = \frac{\lambda_2}{dx_2} + \frac{\lambda_2}{dx_2}$$
$$b_P = q_{v2} \frac{V_2}{A}$$

e) $i = NB + 1$

$$Q_W - Q_{conv} + Q_{v2} = 0$$
$$-\lambda_2 \left(\frac{T_P - T_W}{d_{WP}}\right) A - \alpha_B (T_P - Tg_A) A + q_{v2} V_2 = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = \frac{\lambda_2}{dx_2}$$
$$a_E = 0$$
$$a_P = \alpha_B + \frac{\lambda_2}{dx_2}$$
$$b_P = q_{v2}\frac{V_2}{A} + \alpha_B T g_B$$

Algorithm

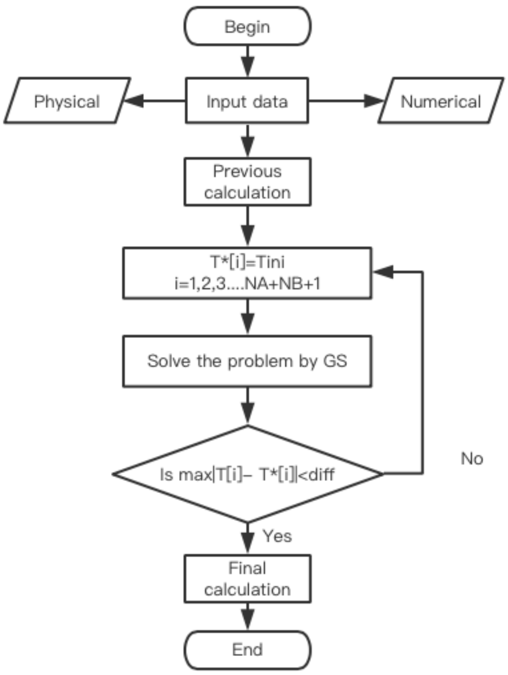

Figure 7 Algorithm of composed wall problem

4.2.4. Results

The numerical method is developed by a Matlab code following the algorithm above and temperature distribution along the wall is obtained, showing in Figure 8. And a new plot (Figure 9) is presented to show temperature distribution in material B with a larger order of magnitude.

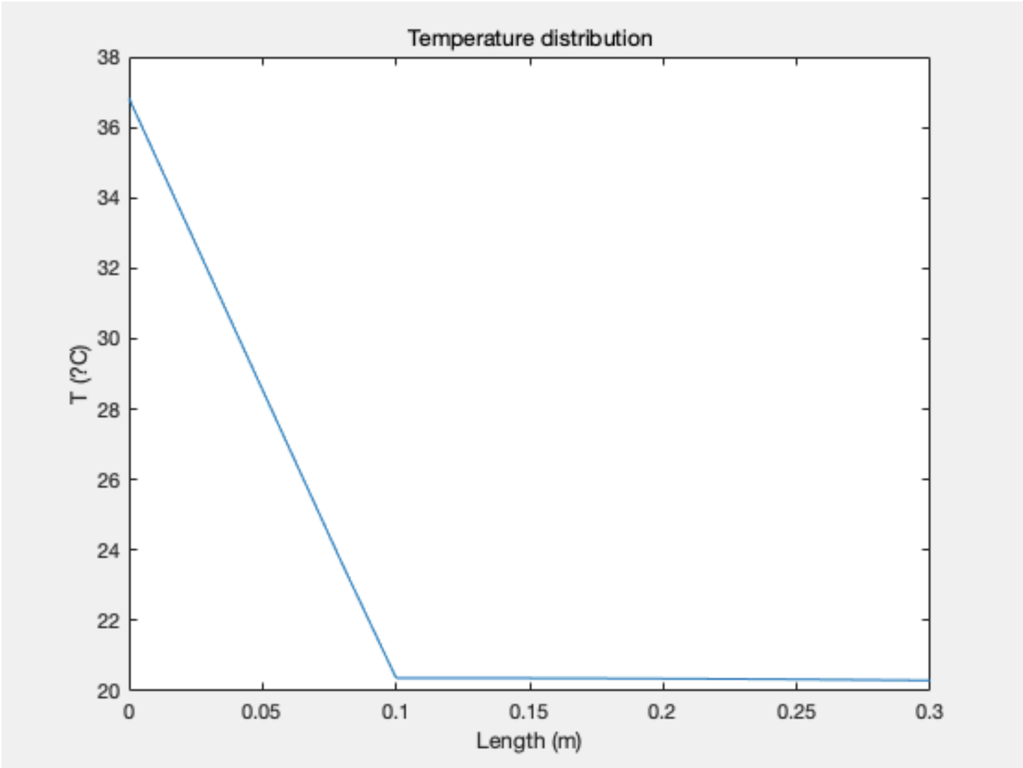

Figure 8 Temperature distribution in composed wall

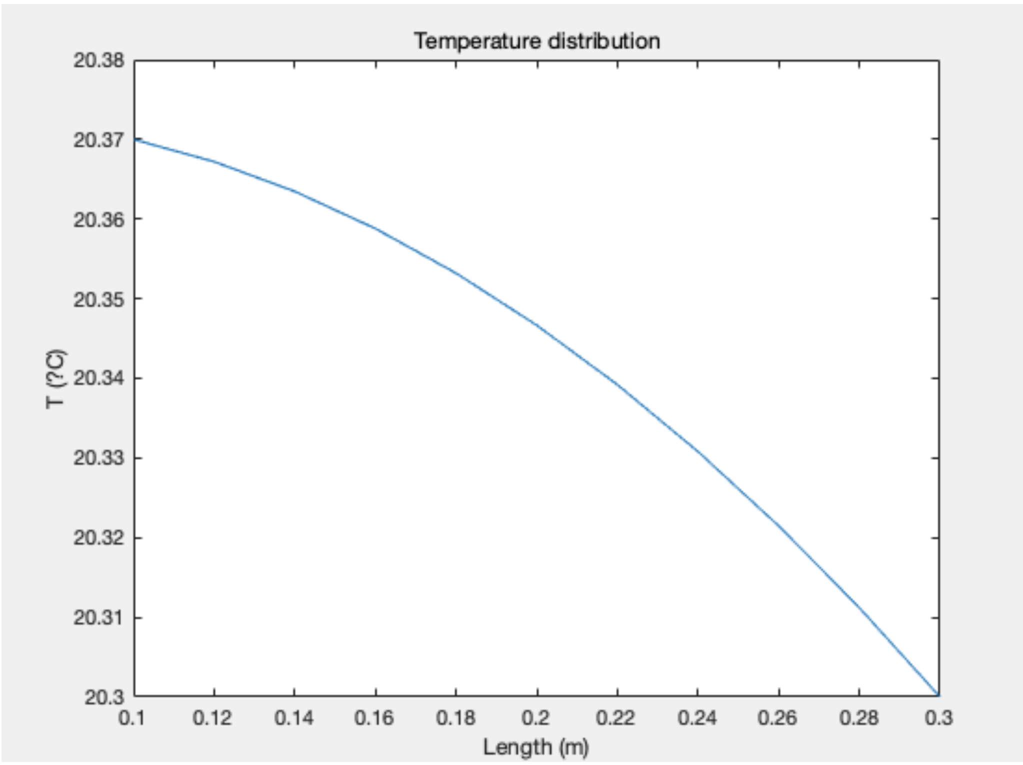

Figure 9 Temperature distribution in material B

Final results are calculated by both analytical and numerical methods, listed in the following table.

	Analytical	Numerical	Differences
T_{w1} (°C)	36.72	36.83	0.29%
T_{w1-2}(°C)	20.30	20.37	0.34%
T_{w2}(°C)	20.27	20.30	0.14%

As we could see the results in the table, there exists very tiny differences between values of these temperatures at the wall which are evaluated by analytical and numerical methods respectively. From the final results, we could draw conclusion that numerical method could be as accurate as analytical method in this case.

4.2.5. Grid sensitivity

A grid sensitivity analysis is a determined study in numerical method to help find the most appropriate meshes. The previous results obtained by numerical method are based on 20+10 nodes, which means the material A is divided into 24 nodes and material B is divided into 12 nodes. As we know that the results from numerical depends on various factors. In order to find the best parameters for the methods, a grid sensitivity study is made.

The reference case is of 20+10 nodes while in other cases, some other combinations are used: 10+5 and 100+50. Same codes are used to get answers and new results came out.

Case of 10+5 nodes

	Analytical	Numerical	Differences
T_{w1} (°C)	36.72	36.95	0.62%
T_{w1-2}(°C)	20.30	20.34	0.19%
T_{w2}(°C)	20.27	20.29	0.09%

Case of 100+50 nodes

	Analytical	Numerical	Differences
T_{w1} (°C)	36.72	36.72	0
T_{w1-2}(°C)	20.30	20.59	0.14%
T_{w2}(°C)	20.27	20.43	0.78%

Comparison of different combination of nodes

	10+5	20+10	100+50
T_{w1} (°C)	0.62%	0.29%	0
T_{w1-2}(°C)	0.19%	0.34%	0.14%
T_{w2}(°C)	0.09%	0.14%	0.78%

As is shown in above tables, generally the final results are close the analytical answers with some tiny differences in both new combinations. Maybe there exist a combination that match analytical results perfectly but it's acceptable to using these combinations. If more accurate results are required, more work need to be devoted.

4.2.6. Parametric study

Apart from grid sensitivity, a parametric study is also of great significance in analysis of its performance. Generally, those input parameters could be reset are boundary conditions, themrophsical properties, geometry conditions and so on. In the following part, three other cases will be presented.

In the basic case, there is an internal heat source in material B and in material A there isn't. What will the new temperature distribution be if the material A is equipped with a heat source or both them are moved heat source?

Case of heat source in both materials

To solve this case, some equations from analytical method in the previous part will be:

The temperature distribution in material A will be:

$$T_1(x) = -\frac{q_{v1}}{2\lambda}x(x-e_1) + \frac{T_{W1-2}-T_{W1}}{e_1}x + T_{W1}$$

The temperature distribution in material B remains the same:

$$T_2(x) = -\frac{q_{v2}}{2\lambda}(x-e_1)(x-e_1-e_2) + \frac{T_{W2}-T_{W1-2}}{e_2}(x-e_1) + T_{W1-2}$$

When apply them into energy conservation at boundary points, equations change correspondingly.

$$\alpha_A(Tg_A - T_{w1}) = -\frac{q_{v1}}{2}e_1 - \lambda_1\frac{T_{w1-2}-T_{w1}}{e_1}$$

$$\alpha_B(T_{w2} - Tg_B) = \frac{q_{v2}}{2}e_2 - \lambda_2\frac{T_{w2}-T_{w1-2}}{e_2}$$

$$\frac{q_{v1}}{2}e_1 - \lambda_1\frac{T_{w1-2}-T_{w1}}{e_1} = -\frac{q_v}{2}e_2 - \lambda_2\frac{T_{w2}-T_{w1-2}}{e_2}$$

When numerical method is used, the discretization equations in material A must be modified:

a) $i = 1$

$$Q_{conv} - Q_E + Q_{v1} = 0$$
$$\alpha_A(Tg_A - T_P)A + \lambda_1\left(\frac{T_E - T_P}{d_{PE}}\right)A + q_{v1}V_1 = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = 0$$
$$a_E = \frac{\lambda_1}{dx_1}$$
$$a_P = \alpha_A + \frac{\lambda_1}{dx_1}$$
$$b_P = \alpha_A Tg_A + q_{v1}\frac{V_1}{A}$$

b) $i = 2\ to\ NA$

$$Q_W - Q_e + Q_{v1} = 0$$
$$-\lambda_1\left(\frac{T_P - T_W}{d_{WP}}\right)A + \lambda_1\left(\frac{T_E - T_P}{d_{PE}}\right)A + q_{v1}V_1 = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = \frac{\lambda_1}{dx_1}$$
$$a_E = \frac{\lambda_1}{dx_1}$$
$$a_P = \frac{\lambda_1}{dx_1} + \frac{\lambda_1}{dx_1}$$
$$b_P = q_{v1}\frac{V_1}{A}$$

c) $i = NA + 1$

$$Q_W - Q_e + Q_{v1} + Q_{v2} = 0$$
$$-\lambda_1\left(\frac{T_P - T_W}{d_{WP}}\right)A + \lambda_2\left(\frac{T_E - T_P}{d_{PE}}\right)A + q_{v1}V_1 + q_{v2}V_2 = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = \frac{\lambda_1}{dx_1}$$
$$a_E = \frac{\lambda_2}{dx_2}$$
$$a_P = \frac{\lambda_1}{dx_1} + \frac{\lambda_2}{dx_2}$$

$$b_P = q_{v1}\frac{V_1}{A} + q_{v2}\frac{V_2}{A}$$

Final results are listed in table below and temperature shows in Figure 10.

	Analytical	Numerical	Differences
T_{w1} (°C)	38.93	39.14	0.51%
T_{w1-2}(°C)	20.54	20.41	0.63%
T_{w2}(°C)	20.26	20.34	0.39%

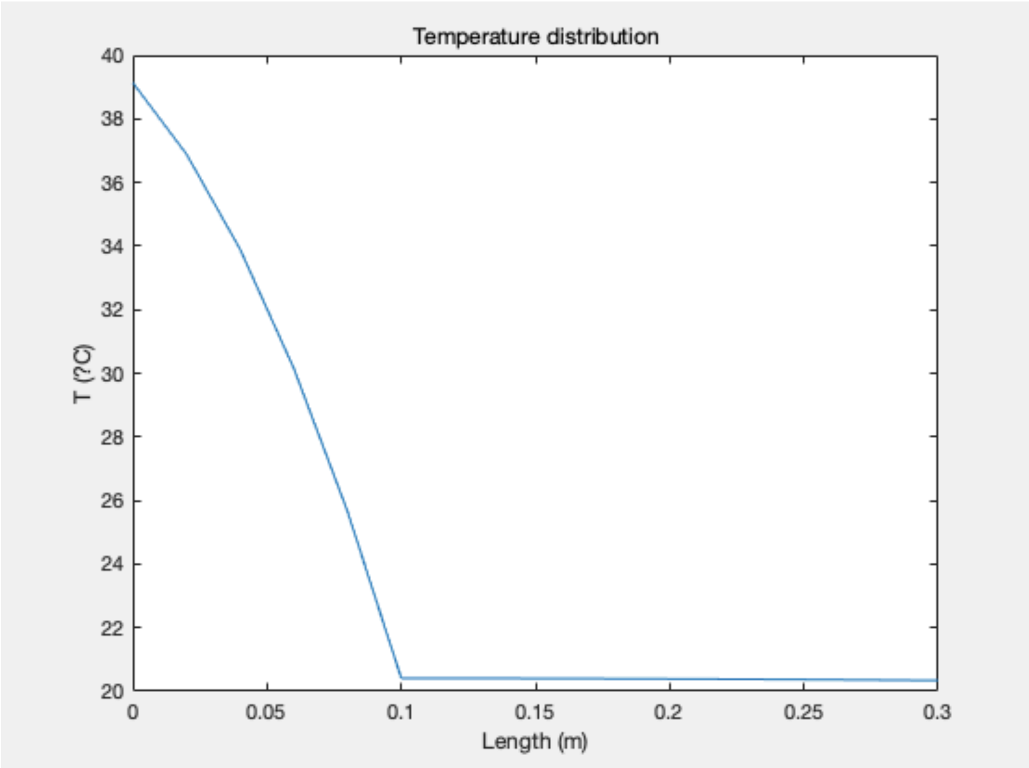

Figure 10 Temperature distribution in composed wall with heat source

Case of no heat source

As for the case of no internal source will be much easier. Same procedures are done as following:

The temperature distribution in material A will be:

$$T_1(x) = \frac{T_{w1-2} - T_{W1}}{e_1}x + T_{W1}$$

The temperature distribution in material B remains the same:

$$T_2(x) = \frac{T_{W2} - T_{w1-2}}{e_2}(x - e_1) + T_{W1-2}$$

When apply them into energy conservation at boundary points, equations change correspondingly.

$$\alpha_A(Tg_A - T_{w1}) = -\lambda_1 \frac{T_{w1-2} - T_{w1}}{e_1}$$

$$\alpha_B(T_{w2} - Tg_B) = -\lambda_2 \frac{T_{w2} - T_{w1-2}}{e_2}$$

$$-\lambda_1 \frac{T_{w1-2} - T_{w1}}{e_1} = -\lambda_2 \frac{T_{w2} - T_{w1-2}}{e_2}$$

However, in this case, discretization equations in material B need to be changed as basic case:

a) $i = NA + 1$

$$Q_W - Q_e + Q_{v1} + Q_{v2} = 0$$
$$-\lambda_1\left(\frac{T_P - T_W}{d_{WP}}\right) + \lambda_2\left(\frac{T_E - T_P}{d_{PE}}\right) = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = \frac{\lambda_1}{dx_1}$$
$$a_E = \frac{\lambda_2}{dx_2}$$
$$a_P = \frac{\lambda_1}{dx_1} + \frac{\lambda_2}{dx_2}$$
$$b_P = 0$$

b) $i = NA + 1\ to\ NB$

$$Q_W - Q_e + Q_{v2} = 0$$
$$-\lambda_2\left(\frac{T_P - T_W}{d_{WP}}\right) + \lambda_2\left(\frac{T_E - T_P}{d_{PE}}\right) = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = \frac{\lambda_2}{dx_2}$$
$$a_E = \frac{\lambda_2}{dx_2}$$
$$a_P = \frac{\lambda_2}{dx_2} + \frac{\lambda_2}{dx_2}$$

$$b_P = 0$$

c) $i = NB + 1$

$$Q_W - Q_{conv} + Q_{v2} = 0$$
$$-\lambda_2 \left(\frac{T_P - T_W}{d_{WP}}\right) - \alpha_B (T_P - Tg_A) = 0$$
$$a_P T_P = a_W T_W + a_E T_E + b_P$$

Where:

$$a_W = \frac{\lambda_2}{dx_2}$$
$$a_E = 0$$
$$a_P = \alpha_B + \frac{\lambda_2}{dx_2}$$

$$b_P = \alpha_B Tg_B$$

Final results are listed in table below and temperature shows in Figure 11.

	Analytical	Numerical	Differences
T_{w1} (°C)	38.06	38.22	0.42%
T_{w1-2}(°C)	20.31	20.22	0.45%
T_{w2}(°C)	20.17	20.17	0

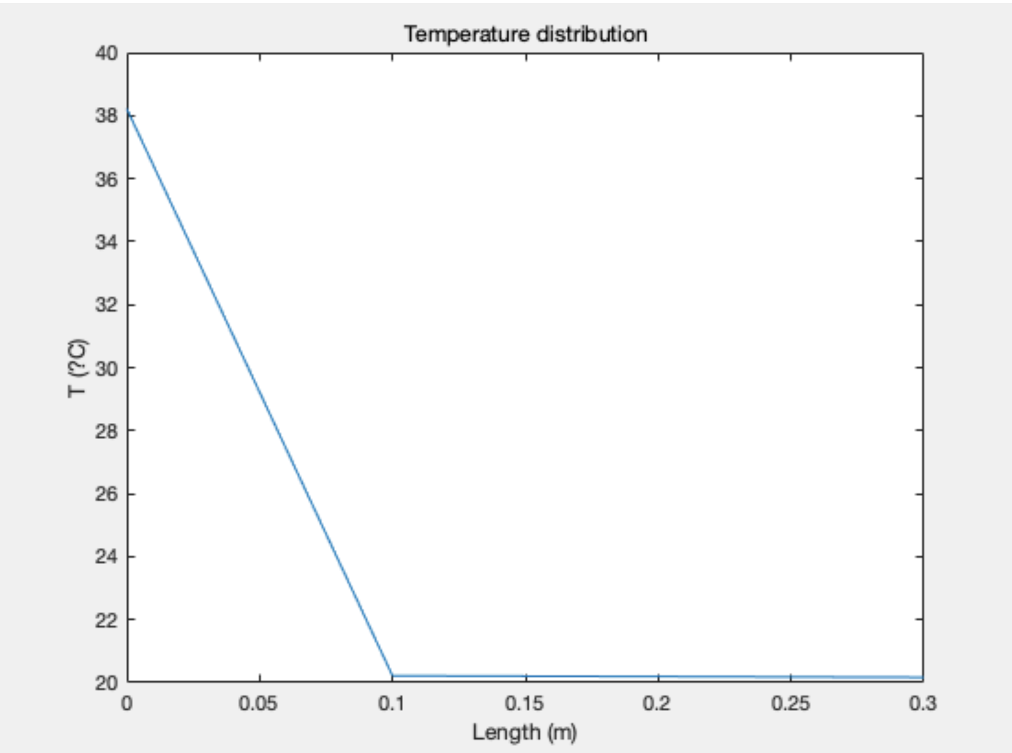

Figure 11 Temperature distribution in composed wall without heat source

Comparison of analytical results

	Heat source in B	Heat source in A+B	No heat source
T_{w1} (°C)	36.72	38.93	38.06
T_{w1-2}(°C)	20.30	20.54	20.31
T_{w2}(°C)	20.27	20.26	20.17

In the case of heat source in B, T_{w1} and T_{w1-2} are the lowest among three cases while T_{w2} isn't due to its internal heat source.

In the case of heat source in both materials, both T_{w1} and T_{w1-2} are the higher than previous case.

In the case of no heat source in either of material, T_{w2} is the lowest one because there is no heat source in material B compared with other cases.

Besides, the geometry conditions also affect the trend of temperature. In basic case, the width of material B is twice of the width of material A. What happens if they're of same width of what material A is thicker?

	$e_1(m) = 0.1$	$e_1(m) = 0.1$	$e_1(m) = 0.2$	$e_1(m) = 0.2$
	$e_2(m) = 0.2$	$e_2(m) = 0.1$	$e_2(m) = 0.2$	$e_2(m) = 0.1$
T_{w1} (°C)	36.83	38.23	38.27	39.07
T_{w1-2}(°C)	20.37	20.39	20.41	20.44
T_{w2}(°C)	20.30	20.33	20.33	20.36

When keep width of material A the same, larger width of material B leads to lower temperatures. However, things are different when it comes to material B where larger width of material A leads to higher temperatures.

5. Convection heat transfer problems

In this chapter, the main focus is convection heat transfer. Like previous chapter, this one is also beginning with a discretized equation but of the convection-diffusion equation and also solve a convection heat transfer problem.

The first objective of this chapter is to present discretized convection-diffusion equation and introduce some schemes to evaluate convective terms and the second goal is to solve a convection heat transfer problem (isothermal tube) using step-by-step method numerically.

5.1. Convection-diffusion equation

The generic convection-diffusion transport equation can be written [10]:

$$\frac{\partial(\rho\emptyset)}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}\emptyset) = \nabla\cdot(\Gamma_\emptyset\nabla\emptyset) + s_\emptyset$$

Where, $\Gamma_\emptyset$ is the diffusion coefficient and $s_\emptyset$ are the source terms.

The mass conservation equation $\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{v}) = 0$ could also be written into the form of convection-diffusion equation.

Where, $\emptyset = 1$, $\Gamma = 0$, $s_\emptyset = 0$.

Similarly, the momentum and energy conservation equations could also be written in this way:

Equation	$\emptyset$	Γ	$s_\emptyset$
Mass	1	0	0
Momentum	$\boldsymbol{v}$	μ	$\nabla\cdot(\boldsymbol{\tau} - \mu\nabla\boldsymbol{v}) - \nabla \mathrm{p} + \rho\mathbf{g}$
Energy	T	$\frac{\lambda}{c_v}$	$\frac{1}{c_v}(-\nabla\cdot\boldsymbol{q}^R - p\nabla\cdot\boldsymbol{v} + \boldsymbol{\tau}:\nabla\boldsymbol{v})$

Finite volume approach is utilized to discretize the above equations and discretization meshes could be structured or unstructured. Time discretization is shown as Figure 12.

Figure 12 Discretization of finite volume approach

The integral from of continuity equation is written as:

$$\frac{\partial}{\partial t}\int_{V_P} \rho dV + \int_{S_f} \rho\vec{v}\cdot\vec{n}dS = 0$$

Integrating between the instants t=n and t=n+1, the previous equation will be

$$\int_{t^n}^{t^{n+1}}\int_{V_P}\frac{\partial}{\partial t}\rho dVdt + \int_{t^n}^{t^{n+1}}\int_{S_f}\rho\vec{v}\cdot\vec{n}dSdt = 0$$

Using implicit scheme to discretize and replacing superindex n with o while n+1 dropped for convenience in a 2D case, the new continuity equation is obtained:

$$\frac{\rho_P - \rho_P^o}{\Delta t}V_P + \dot{m}_e - \dot{m}_w + \dot{m}_n - \dot{m}_s = 0$$

As for discretization of convection-diffusion equation, each term using implicit scheme could be presented:

Unsteady terms: $\int_{t^n}^{t^{n+1}}\int_{V_P}\frac{\partial(\rho\emptyset)}{\partial t}dVdt \approx V_P(\rho_P\emptyset_P - \rho_P^o\emptyset_P^o)$

Convective terms: $\int_{t^n}^{t^{n+1}}\int_{V_P}\nabla\cdot(\rho\vec{v}\emptyset)dVdt \approx (\dot{m}_e\emptyset_e - \dot{m}_w\emptyset_w + \dot{m}_n\emptyset_n - \dot{m}_s\emptyset_s)\Delta t$

Diffusion terms: $\int_{t^n}^{t^{n+1}}\int_{V_P}\nabla\cdot(\Gamma_\emptyset\nabla_\emptyset)dVdt \approx (D_e(\emptyset_E - \emptyset_P) - D_w(\emptyset_P - \emptyset_W) + D_n(\emptyset_N - \emptyset_P) - D_s(\emptyset_P - \emptyset_S))\Delta t$

Where, $D_e = \Gamma_e S_e/d_{PE}$, $D_w = \Gamma_w S_w/d_{PW}$, $D_n = \Gamma_n S_n/d_{PN}$ and $D_s = \Gamma_s S_s/d_{PS}$

Source terms: $\int_{t^n}^{t^{n+1}}\int_{V_P}S_\emptyset dVdt \approx (S_C^\emptyset + S_P^\emptyset\emptyset_P)V_P\Delta t$

A new equivalent form of above terms using discretized continuity equation (mass conservation equation) could be written as:

$$\rho_P^o \frac{\emptyset_P - \emptyset_P^o}{\Delta t} V_P + \dot{m}_e(\emptyset_e - \emptyset_p) - \dot{m}_w(\emptyset_w - \emptyset_p) + \dot{m}_n(\emptyset_n - \emptyset_p) - \dot{m}_s(\emptyset_s - \emptyset_p)$$

$$= D_e(\emptyset_E - \emptyset_P) - D_w(\emptyset_P - \emptyset_W) + D_n(\emptyset_N - \emptyset_P) - D_s(\emptyset_P - \emptyset_S) + (S_C^{\emptyset} + S_P^{\emptyset}\emptyset_P)V_P$$

The next step is to evaluate convective terms [11]. Central-difference scheme (CDS) and Upwind-difference scheme (UDS) are the simplest finite difference methods and widely used ones. Apart from them, Exponential-difference scheme (EDS), Second-order upwind scheme (SUDS) and Quadratic upwind interpolation for convective kinematics (QUICK) could also be applied into some specific cases.

CDS assumes a linear interpolation between the nodes.

$$\emptyset_e - \emptyset_P = f_e(\emptyset_E - \emptyset_P)$$

$$f_e = \frac{d_{Pe}}{d_{PE}}$$

UDS assumes that the convected value at the cell face is adapted from upstream node.

$$\emptyset_e - \emptyset_P = f_e(\emptyset_E - \emptyset_P)$$

$$f_e = \begin{cases} 0, & if\ \dot{m}_e > 0 \\ 1, & if\ \dot{m}_e < 0 \end{cases}$$

EDS's assumption is based on a simplified convection-diffusion equation. For instance, a two-dimensional case without source term:

$$\frac{d}{dx}(\rho v_x \phi) = \frac{d}{dx}(\Gamma \frac{d\phi}{dx})$$

Considering ρ, v_x, and Γ are constant between nodal values and equal to the one at the face, so the integrated could be obtained easily.

$$\emptyset_e - \emptyset_P = f_e(\emptyset_E - \emptyset_P)$$

$$f_e = \frac{e^{Pe\, d_{Pe}/d_{PE}} - 1}{e^{Pe} - 1}$$

SUDS is designed when second-order accuracy is desired. In this approach, three points are used instead of 2: two nodes at upwind of face and one at downwind.

QUICK is also a higher-order differencing scheme that considers a three-point upstream weighted quadric interpolation foe cell face values. To find the cell face value, a quadratic function passing through two bracketing or surrounding nodes and one node on the upstream side are needed.

In order to simplify the representation of high-order schemes, normalization of variables is introduced. Located at the face position, D refers to the downstream node, C is the first upstream node and U is the most upstream node. Then the dependent variable and position could be written in this way:

$$\hat{x} = \frac{x - x_U}{x_D - x_U}$$

$$\widehat{\emptyset} = \frac{\emptyset - \emptyset_U}{\emptyset_D - \emptyset_U}$$

These schemes mentioned above can be presented in the following table after normalizing:

Scheme	Face Value
CDS	$\widehat{\emptyset}_f = \frac{\hat{x}_f - \hat{x}_C}{1 - \hat{x}_C} + \frac{\hat{x}_f - 1}{\hat{x}_C - 1}\widehat{\emptyset}_C$
UDS	$\widehat{\emptyset}_f = \widehat{\emptyset}_C$
SUDS	$\widehat{\emptyset}_f = \frac{\hat{x}_f}{\hat{x}_C}\widehat{\emptyset}_C$
QUICK	$\widehat{\emptyset}_f = \hat{x}_f + \frac{\hat{x}_f(\hat{x}_f - 1)}{\hat{x}_C(\hat{x}_C - 1)} + (\widehat{\emptyset}_C - \hat{x}_C)$

Some of first order schemes are easy to introduce to the discretized convection-diffusion equation to get final form of equation while high resolution (HRS) schemes are suggested to use deferred correction approach due to their large molecules. The basic idea is:

$$\emptyset_f^{HRS} - \emptyset_P = \left(\emptyset_f^{UDS} - \emptyset_P\right) + (\emptyset_f^{HRS,*} - \emptyset_f^{UDS,*})$$

UDS written in this way:

$$\dot{m}_e(\emptyset_e^{UDS} - \emptyset_P) = \frac{\dot{m}_e - |\dot{m}_e|}{2}(\emptyset_E - \emptyset_P)$$

Using the deferred correction approach like equation above, the final form of convection-diffusion equation is

$$\rho_P^o \frac{\emptyset_P - \emptyset_P^o}{\Delta t} V_P + \frac{\dot{m}_e - |\dot{m}_e|}{2}(\emptyset_E - \emptyset_p) - \frac{\dot{m}_w + |\dot{m}_w|}{2}(\emptyset_W - \emptyset_p) + \frac{\dot{m}_n - |\dot{m}_n|}{2}(\emptyset_N - \emptyset_p$$

$$-\frac{\dot{m}_s + |\dot{m}_s|}{2}(\emptyset_S - \emptyset_p)$$

$$= D_e(\emptyset_E - \emptyset_P) - D_w(\emptyset_P - \emptyset_W) + D_n(\emptyset_N - \emptyset_P) - D_s(\emptyset_P - \emptyset_S) - \dot{m}_e(\emptyset_e^{HRS,*} - \emptyset_e^{UDS,*})$$

$$+\dot{m}_w(\emptyset_w^{HRS,*} - \emptyset_w^{UDS,*}) - \dot{m}_n(\emptyset_n^{HRS,*} - \emptyset_n^{UDS,*}) + \dot{m}_s(\emptyset_s^{HRS,*} - \emptyset_s^{UDS,*}) + (S_C^{\emptyset} + S_P^{\emptyset}\emptyset_P)V_P$$

There are two types of boundary conditions when solving ordinary or partial differential equations: Dirichlet BC and Neumann BC.

In the Dirichlet BC condition, the variable itself is known at the boundary. Fluxes could be calculated:

$$j_e = -\Gamma_P \frac{\emptyset_E - \emptyset_P}{d_{PE}}$$

In the Neumann BC condition, fluxes are known at the boundary. The variable could be calculated:

$$\emptyset_E = \emptyset_P - \frac{j_E d_{PE}}{\Gamma_P}$$

5.2. Problem of isothermal tube

5.2.1. Problem description

The problem is a case combined forced convection and heat conduction phenomena. A fluid flows inside an isothermal tube of internal diameter D_i and length L. The inlet conditions are known: v_{in}, p_{in} and T_{in}.

The objective is to evaluate the average exit velocity, pressure and temperature: v_{out}, p_{out} and T_{out}. The total heat flux exchanged between the wall and fluid, $\dot{Q_W}$ must also be calculated.

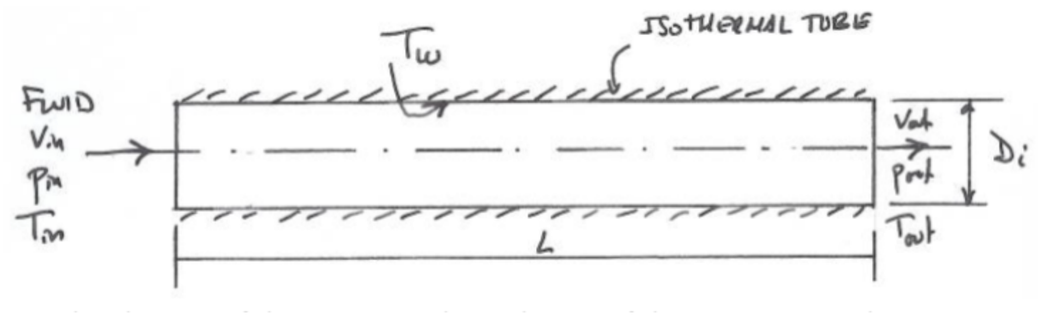

Figure 13 Isothermal tube [9]

Fluid in case 1 is water, case 2 is thermal oil and case 3 is air. The other initial conditions are listed in the following table.

Geometry condition	
$D_I(m)$	0.02
$L(m)$	5
$v_{in}(m/s)$	1(case1 and 2)
$v_{in}(m/s)$	30(case3)
$T_{in}(°C)$	20
$P_{in}(pa)$	2.026e5
$T_w(°C)$	95

5.2.2. Numerical method

Before solving the problem numerically, some topics needs to discussed previously. The case is assumed as a one-dimensional steady case and mean values are used at each cross-sectional area as well as local heat transfer coefficients.

The first step to solve the problem using numerical method is to discretize the domain.

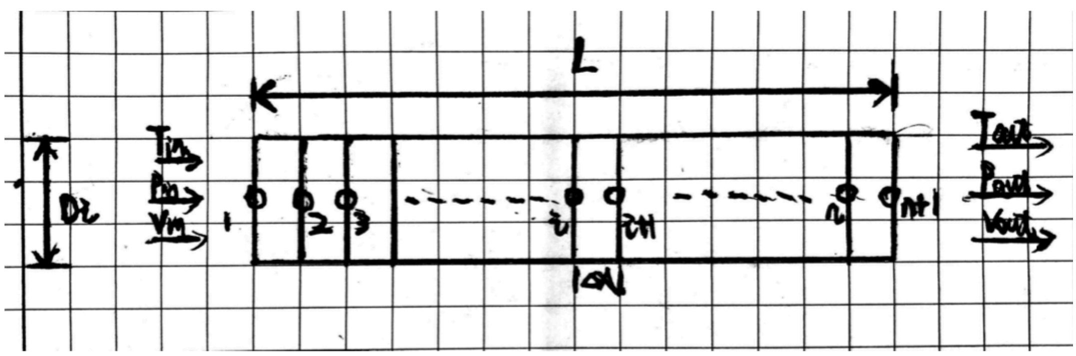

Figure 14 Discretization of domain

As Figure 14 shows, the isothermal could be divided into n control volumes and n+1 nodes. The method applied in this case is step by step method which is to calculate outlet temperature, pressure and velocity from inlet conditions using conservative equations (mass, momentum and energy). The outlet properties of a forward node are the inlet conditions for the backward one. In this way, the fluid properties from first node to last one could be obtained.

Mass conservation

$$\int \rho\, \vec{v}\, \vec{n}\, ds = 0$$

$$\downarrow$$

$$\dot{m}[i+1] - \dot{m}[i] = 0$$

$$\downarrow$$

$$\dot{m}[i+1] = \dot{m}[i] = \dot{m} = constant$$

$$\downarrow$$

$$\dot{m} = \dot{m}[1] = \rho[1]v[1]S$$

Where, S refers to cross-sectional surface and is constant

$$S = \frac{\pi}{4} {D_i}^2$$

The density ρ for the fluid only depends on the temperature when it's a liquid while it depends on both temperature and pressure when if it's gas. This equation is called state equation.

$$\rho_g[i] = \frac{P[i]}{R \cdot T[i]}$$

$$\rho_l[i] = f(T[i])$$

The specific function could be found in Formula Section D.

Since the cross-sectional surface S is constant, the mass conservation equation could turn into the following one:

$$\mathrm{v}[i+1] = \frac{\rho[i]}{\rho[i+1]} v[i]$$

Momentum conservation

$$\int v\rho\vec{v}\, \vec{n}\, ds = F_x + B_x$$

Where, F_x refers to superficial force like pressure force and viscous force and B_x is body force which could be neglected in this case. Thus, the equation could be written as:

$$m[i+1]v[i+1] - m[i]v[i] = (P[i] - P[i+1])S - \tau_w \pi \Delta x$$

Where,

$$\Delta x = \frac{L}{N}$$

τ_w is the viscous shear stresses at the wall (N/m^2), it could be evaluated by [13]:

$$\tau_w = \frac{f_i \rho_i {v_i}^2}{2}$$

f_i is the skin friction coefficient that depends on several factors: cross-sectional shape, hydraulic diameter (D_i), Reynolds number (Re) and relative roughness (ε_r).

ρ_i and v_i are average density and velocity in the control volume i using arithmetic mean values of inlet and outlet conditions. The final momentum conservation equation would be:

$$P[i+1] = P[i] - f_i \frac{\rho_i {v_i}^2}{2} \frac{\pi D_i \Delta x}{S} - \frac{\dot{m}}{S}(v[i+1] - v[i])$$

Energy conservation

$$\int (h + e_k + e_k)\rho \vec{v}\,\vec{n}\,ds = Q_w$$

Where, e_k and e_k could be neglected. Then it's easy to obtain the following equation:

$$\dot{m}(h[i+1] - h[i]) = \alpha_i (T_w - T_i)\pi D_i \Delta x$$

At left side of equation, some deductions are made:

$$h[i+1] - h[i] = dh = C_p(T)dT = {C_p}_i(T[i+1] - T[i])$$

The final energy conservation equation is given by:

$$\dot{m}{C_p}_i(T[i+1] - T[i]) = \alpha_i (T_w - T_i)\pi D_i \Delta x$$

In order to obtain α_i heat transfer coefficient (W/m^2K), the thermophysical properties needs to be evaluated first: μ dynamic viscosity (kg/ms), λ thermal conductivity (W/mK) and C_p specific heat (J/kgK) as well as μ_w which refers to the fluid viscosity using the temperature of tube. All these thermophysical properties could be calculated by functions related to the temperature (see formula Section D).

After that, non-dimensional numbers need to calculated: Re, Pr, Gz and Nu [13].

$$Re_i = \frac{\rho_i v_i D_i}{\mu_i}$$

$$Pr_i = \frac{\mu_i Cp_i}{\lambda_i}$$

$$Gz_i = \frac{\pi D_i}{4L} Re_i Pr_i$$

$$Nu_i = CRe_i^{\ m} Pr_i^{\ n} K$$

Where, m,n and K depends on the types of flow and the non-dimensional number.

Finally, the heat transfer coefficient for control volume i could be obtained by [13]:

$$\alpha_i = \frac{Nu_i \lambda_i}{D_i}$$

Algorithm

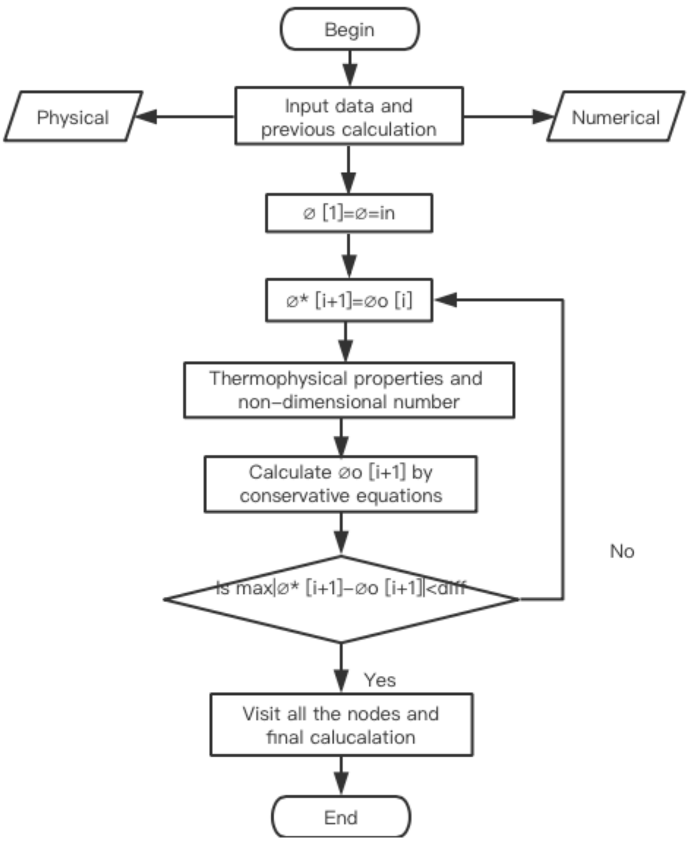

Figure 15 Algorithm of isothermal tube problem

5.2.3. Results

Final results are obtained using step-by-step method and codes are written to develop it. They are collected separately in three cases and the comparisons with others results by analytical methods are made. One thing has to be stated that the number of nodes is 100 in numerical method.

From the data shown in these tables, results obtained by numerical and analytical methods are close in general. In case 1 (water), all the outlet conditions (temperature, velocity, pressure and heat flux) from numerical method are almost equal to the analytical ones with average differences less than 1%. In case 2 (oil) and case 3 (air), there aren't large differences among these outlet conditions. It's easy to draw conclusion that the numerical method performs well in this problem.

Case1 (water)

	Analytical results	Numerical results	Differences
Re	36751	38269	4%
Pr	3.56	3.38	5%
K	1.083	1.096	1.2%
Gz	410	410	0
f	0.0056	0.0056	0
$\alpha(W/m^2K)$	6385	6564	0.01%
$v_{out}(m/s)$	1.025	1.025	0
$p_{out}(pa)$	197143	198747	0.79%
$T_{out}(°C)$	78.74	79.13	0.49%
$Q_W(W)$	77070	77582	0.66%

Case 2 (oil)

	Analytical results	Numerical results	Differences
Re	221	220	0.4%
Pr	1226	1227	0.08%
K	0.246	0.243	1.2%
Gz	850	850	0
f	0.072	0.072	0
$\alpha(W/m^2K)$	173	165	4.6%
$v_{out}(m/s)$	1.005	1.005	0
$p_{out}(pa)$	163371	165844	1.5%
$T_{out}(°C)$	27.74	27.71	0.1%
$Q_W(W)$	359	3845	896.14%

Case 3 (air)

	Analytical results	Numerical results	Differences
Re	72202	62026	14.09%
Pr	0.70	0.71	1.4%
K	0.99	0.99	0
Gz	159	160	0.6%
f	0.0048	0.0050	4.1%
$\alpha(W/m^2K)$	219	230	5.0%
$v_{out}(m/s)$	38.56	39.13	1.4%
$p_{out}(pa)$	193508	193815	0.15%
$T_{out}(°C)$	91.43	91.31	0.13%
$Q_W(W)$	1614	1619	0.30%

Figure 16 shows the temperature distribution of three fluids. The temperature of air rises very quickly and it's the closest to the tube temperature following the temperature of water while the temperature of thermal oil hardly increases.

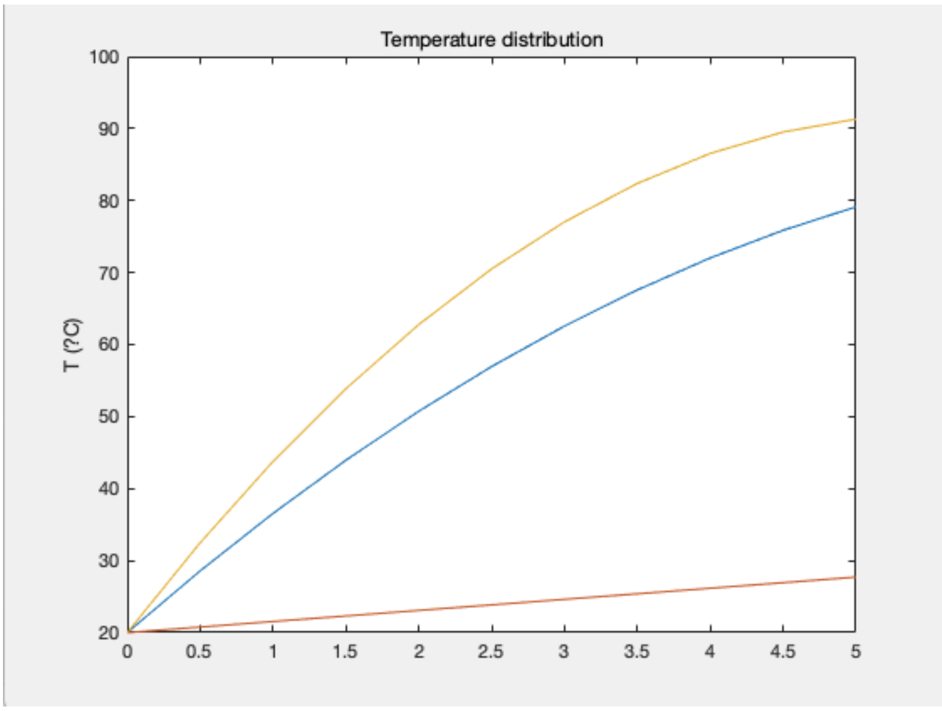

Figure 16 Temperature distribution of three fluids

Figure 17 presents pressure distribution which shows an opposite tendency from temperature. It's easy to find that pressure drop of water and air isn't as high as thermal oil, whose pressure drop is noticeable.

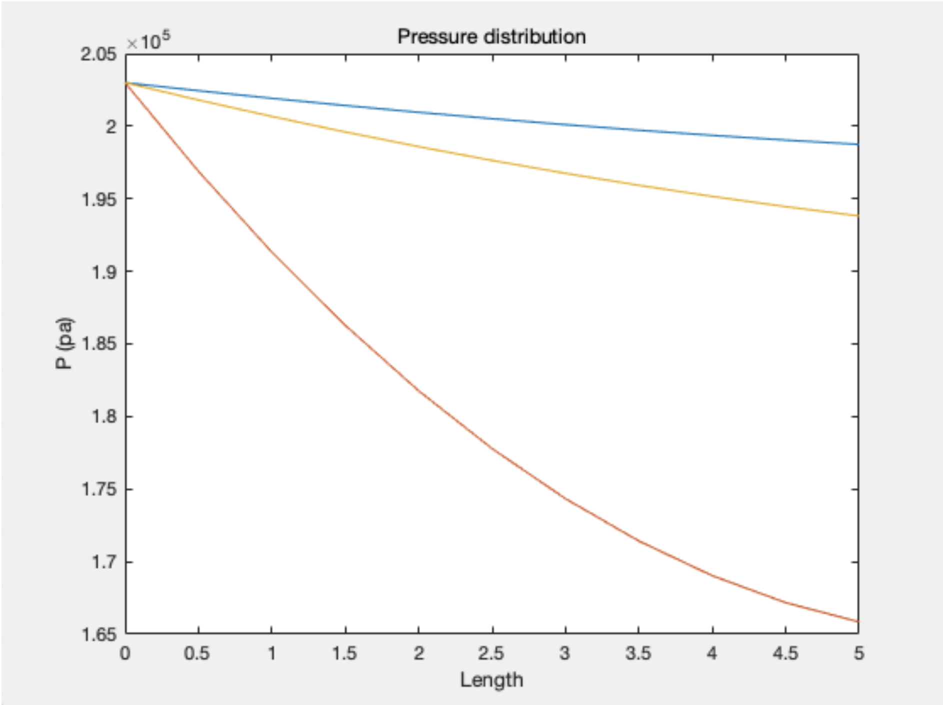

Figure 17 Pressure distribution of three fluids

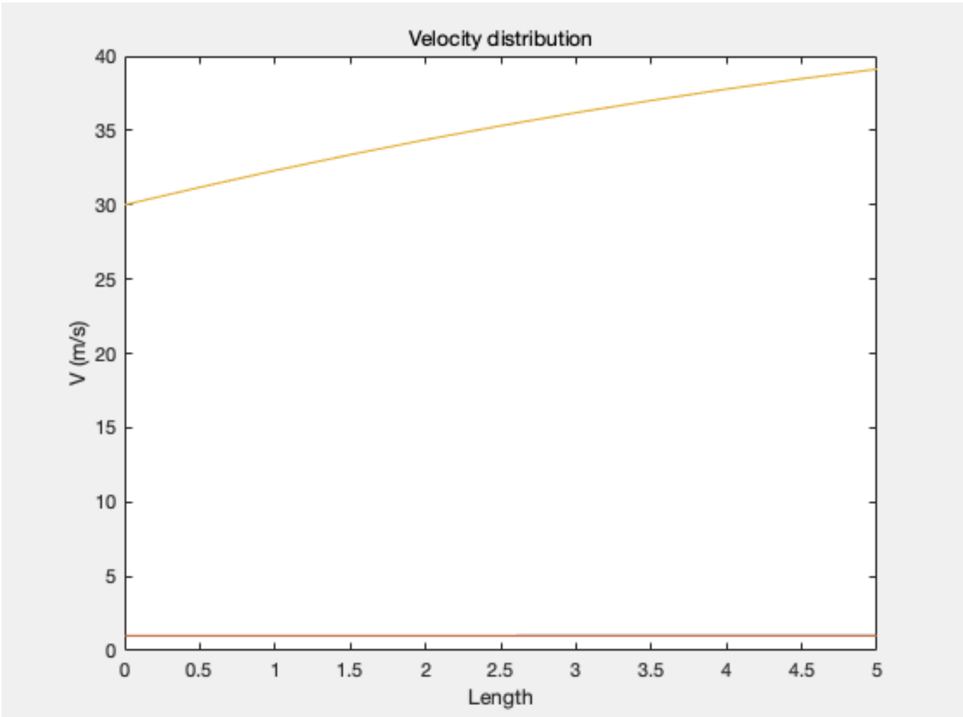

Figure 18 Velocity distribution of three fluids

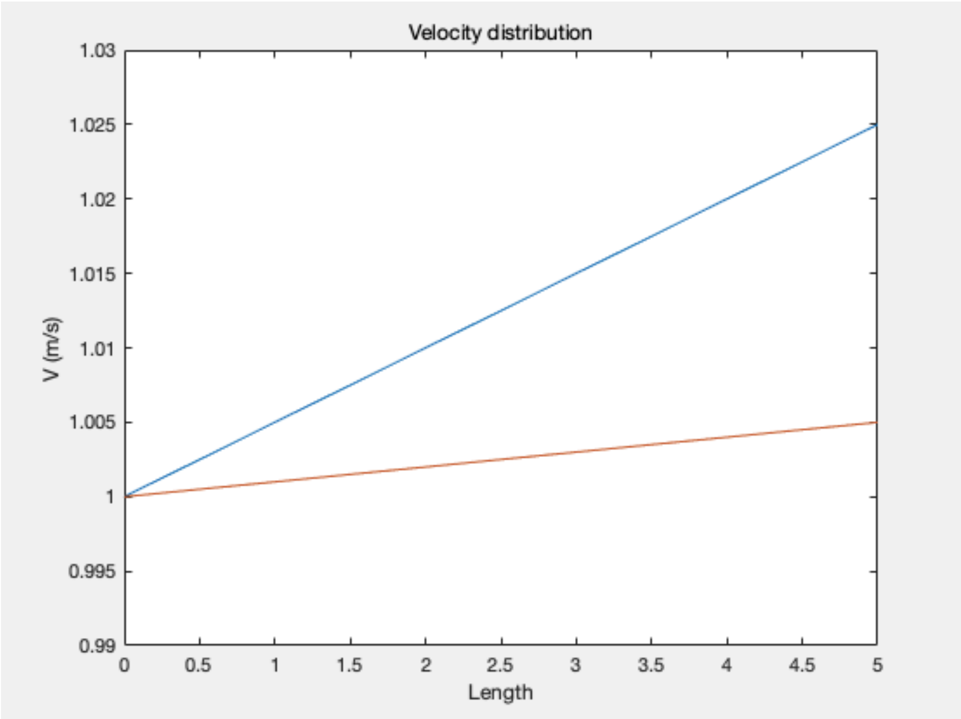

Figure 19 Velocity distribution of water and thermal oil

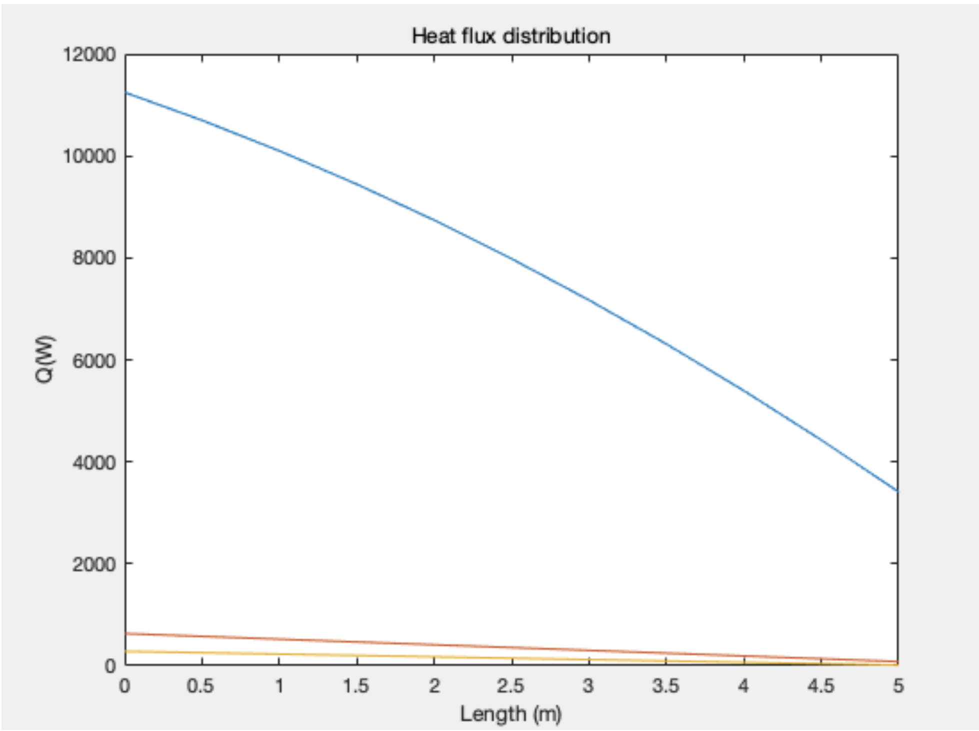

Figure 20 Heat flux distribution of three fluids

Figure 18 shows the variation of velocity for three fluids. The velocity of water and thermal oil nearly remains constant while it changes sharply in the case of air. Figure 19 shows the velocity distribution of water and thermal oil with larger order of magnitude.

Figure 20 shows the heat flux distribution: the heat flux of water reduces sharply while the change of other two fluids is tiny.

5.2.4. Gird sensitivity

Like the conduction case, a parametric study is also made in this problem to find optimized combination. It's undouble that numbers of nodes have large influence on the final results so some new cases with different numbers of nodes (N=1, N=10 and N=100) will be presented to check if N=100 is the best option.

Case of water

	N=1	N=10	N=1000
$v_{out}(m/s)$	1.029	1.026	1.025
$p_{out}(pa)$	199277	199212	198158
$T_{out}(°C)$	79.94	79.51	78.86
$Q_W(W)$	78646	78082	77229

Case of oil

	N=1	N=10	N=1000
$v_{out}(m/s)$	1.017	1.011	1.005
$p_{out}(pa)$	167913	167030	164861
$T_{out}(°C)$	27.17	27.41	27.72
$Q_W(W)$	3574	3695	3850

Case of air

	N=1	N=10	N=1000
$v_{out}(m/s)$	39.72	39.26	38.91
$p_{out}(pa)$	195120	194205	193747
$T_{out}(°C)$	90.33	91.02	91.35
$Q_W(W)$	1596	1612	1620

To compare these results horizontally, these data are reorganized and some new data are obtained to indicate the influence of nodes' number on final answers.

Differences of outlet temperature with different numbers of nodes

	water	oil	air
N=1	1.52%	2.05%	1.20%
N=10	0.98%	1.19%	0.45%
N=100	0.49%	0.10%	0.13%
N=1000	0.15%	0.07%	0.09%

Differences of outlet pressure with different numbers of nodes

	water	oil	air
N=1	1.08%	2.78%	0.83%
N=10	1.05%	2.24%	0.36
N=100	0.79%	1.5%	0.15%
N=1000	0.52%	0.009%	0,01%

Differences of outlet velocity with different numbers of nodes

	water	oil	air
N=1	0.39%	1.19%	3.01%
N=10	0.10%	0.60%	1.82%
N=100	0	0	1.4%
N=1000	0	0	1.0%

After reviewing these tables, we can see that the differences becoming smaller with the increase of nodes and when nodes reach 100, they 're infinitely close to zero. That's to say, when the quantity of nodes is larger than 100, it's not the decisive factor of numerical method. Taken into some other factors like time costs, N=100 can be an optimized option.

5.2.5. Parametric study

Length of tube

In the basic case, the length of isothermal tube is 5 while in next new case, length of tube will be extended to 10m. Using the same algorithm, temperature distribution of three different fluids is obtained (Figure 21). From Figure 22 shows heat flux distribution along the isothermal tube.

Apart from length of tube, diameter of tube is also an important parameter to this kind of problem. In the basic case, the diameter is 20mm and in the next part, case of 10mm will be presented.

	L=5	L=10	differences
water	79.13	92.53	16.93%
oil	27.71	35.44	27.90%
air	91.31	94.81	3.83%

	Di=0.02	Di=0.01	differences
water	79.13	93.57	18.25%
oil	27.71	35.61	28.51%
air	91.31	95.00	4.04%

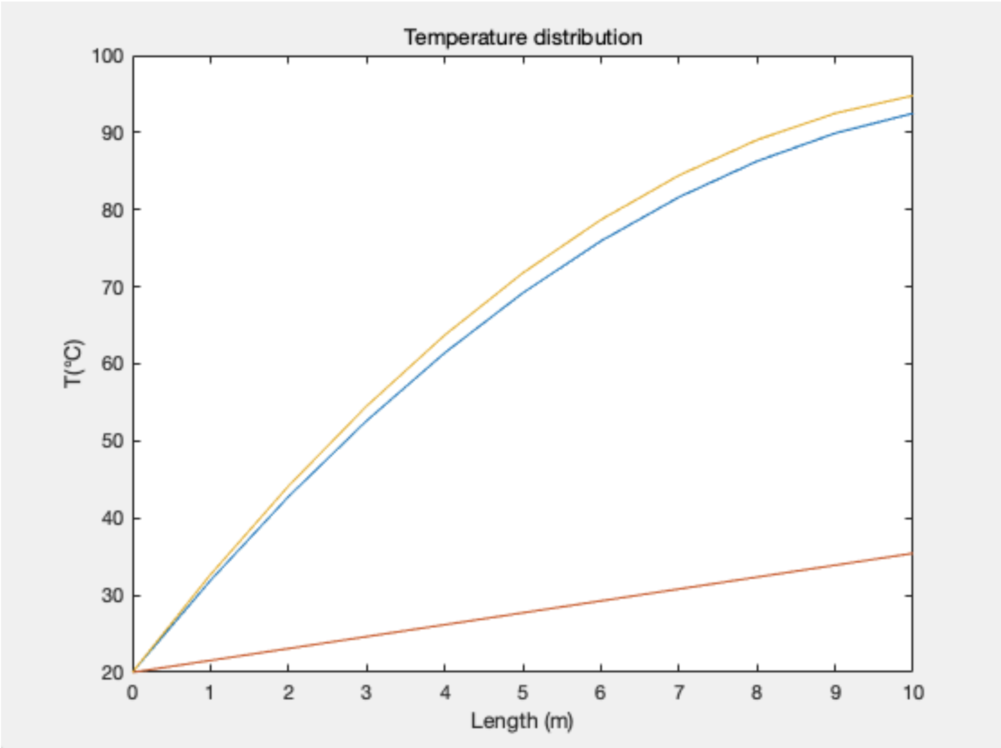

Figure 21 Temperature distribution of three fluids (L=10)

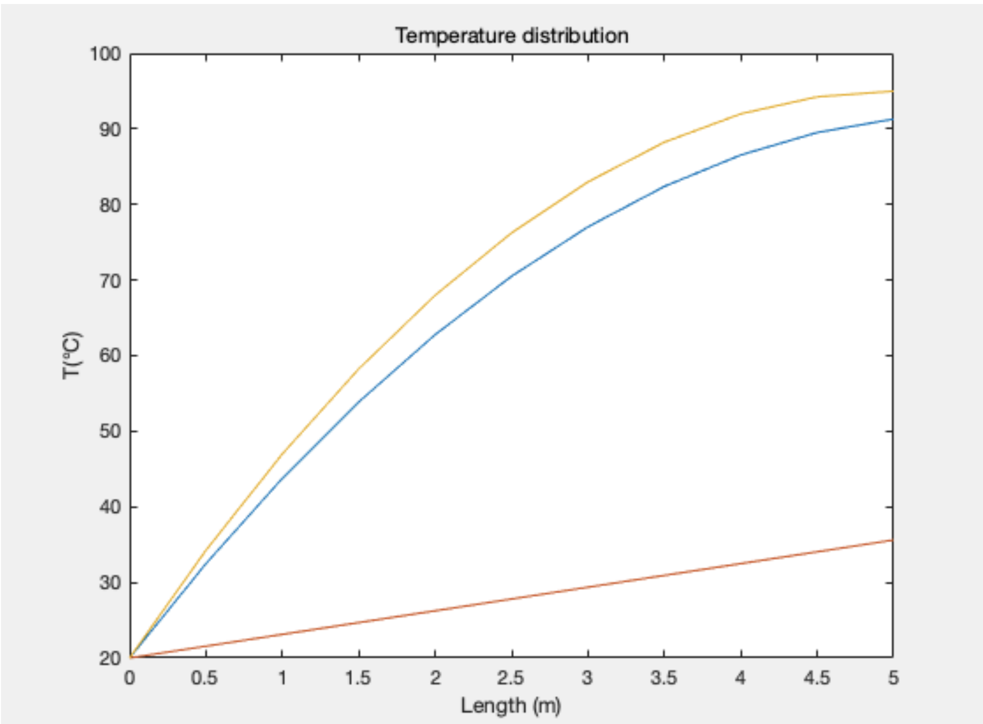

Figure 22 Temperature distribution of three fluids (Di=0.01)

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From the final results, we could see that no matter expanding the length of tube or reducing the diameter of tube, it helps to exchanger heat. Among these fluids, outlet temperature of oil increases sharply with nearly one third following by water and air temperature changes slightly because it’s close to the ambient temperature.

6. Double tube heat exchangers

This chapter will show methodologies to solve double tube heat exchangers for both co-current flow and counter flow using semi-analytical method and numerical method. Then a double tube heat exchanger problem combined different heat transfer mechanisms that explained previously will be solved.

The objective of this chapter is to introduce briefly the methodologies for double tube heat exchangers. For semi-analytical methods, F-factor and e-NTU methods will be presented. Apart from that, the main concentrate will be on solving the problem numerically.

6.1. Methodologies

A double tube heat exchanger is a basic arrangement that consist of two tubes with different parameters where one is concentrically positioned within a larger one. Generally, two fluids flow in same direction called co-current flow or in opposite directions called counter flow (see Figure 23).

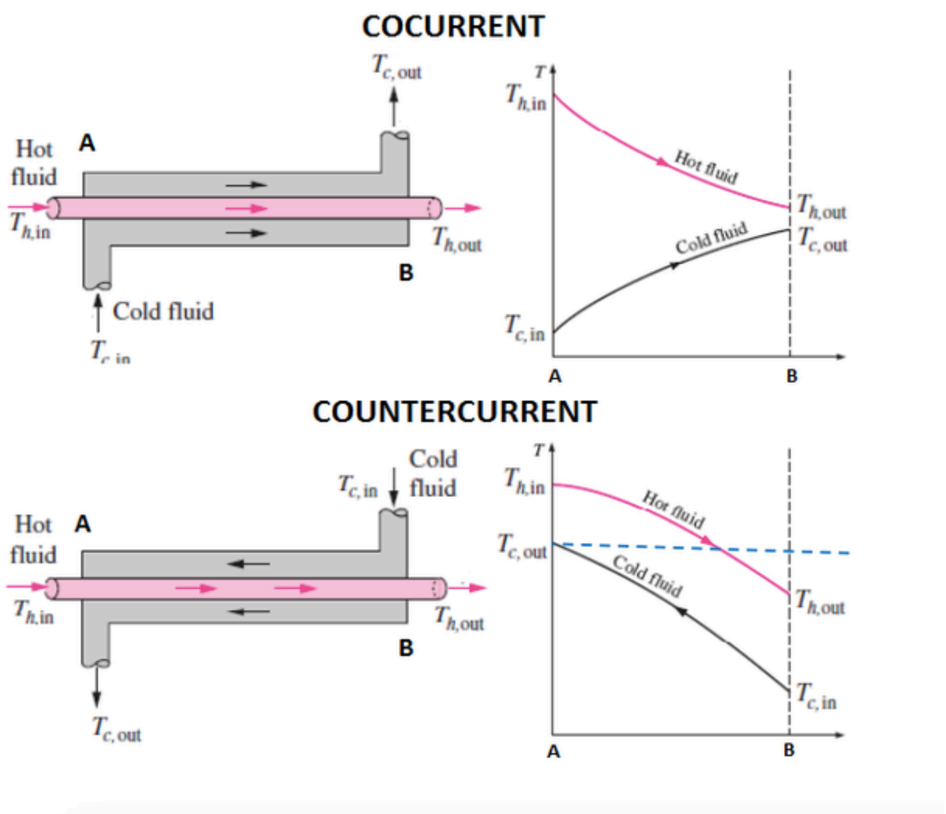

Figure 23 Double tube heat exchanger [14]

To solve the prediction cases of heat exchanger problems, basically there're semi-analytical methods and numerically methods. The general methodology of analytical method is F-Factor [15] method.

F of F-Factor method is defined as:

$$F = \frac{\Delta T_m}{\Delta T_{lm}} \quad (0 < F < 1)$$

It's used as a geometric correlation factor when LMTD ΔT_{lm} (Log Mean Temperature Difference) of a counter flow heat exchanger is applied providing the effective temperature difference. It makes the heat exchanger's departure from ideal behavior of a counter flow heat exchanger have the same terminal temperatures. Log Mean Temperature Difference is defined as:

$$\Delta T_{lm} = \frac{(T_{1i} - T_{2o}) - (T_{1o} - T_{2i})}{\ln \frac{T_{1i} - T_{2o}}{T_{1o} - T_{2i}}}$$

It depends on flux configuration, temperature effectiveness P and heat capacity rate ratio R. P and R are defined as:

$$P = \frac{T_{2o} - T_{2i}}{T_{1i} - T_{1o}}$$

$$R = \frac{\dot{m}_2 Cp_2}{\dot{m}_1 Cp_1} = \frac{C_2}{C_1}$$

Obviously, for counter flow, $F = 1$.

For co-current flow:

$$F = \frac{(1 + R)\ln \left[\frac{1 - P}{1 - PR}\right]}{(1 - R)\ln[1 - P(1 + R)]}$$

Apart from the algebraic equation to calculate F, it's also can be read graphically. As an example, Figure 24 shows mean temperature difference relationships in a co-current flow heat exchanger.

Then for any configuration, we have following equations:

$$\dot{Q} = \dot{m}_1 \dot{Cp_1} (T_{1\iota} - T_{Io})$$

$$\dot{Q} = \dot{m}_2 C\dot{p}_2 (T_{2o} - T_{2\iota})$$

$$\dot{Q} = U_o \Delta T_m A_o = U_o F \Delta T_{lm} A_o$$

Where,

$$F = F(configuration, P, R)$$

$$U_o = \left[\frac{1}{\alpha_{fi}} \frac{P_o}{Pi} + \frac{P_o}{2\pi\lambda_t} \ln\frac{D_o}{D_i} + \frac{1}{\alpha_{fi}} \right]$$

$$\alpha_{fi} = \left(\frac{1}{\alpha_i} + R_{fi} \right)^{-1}, \alpha_{fo} = \left(\frac{1}{\alpha_o} + R_{fo} \right)^{-1}$$

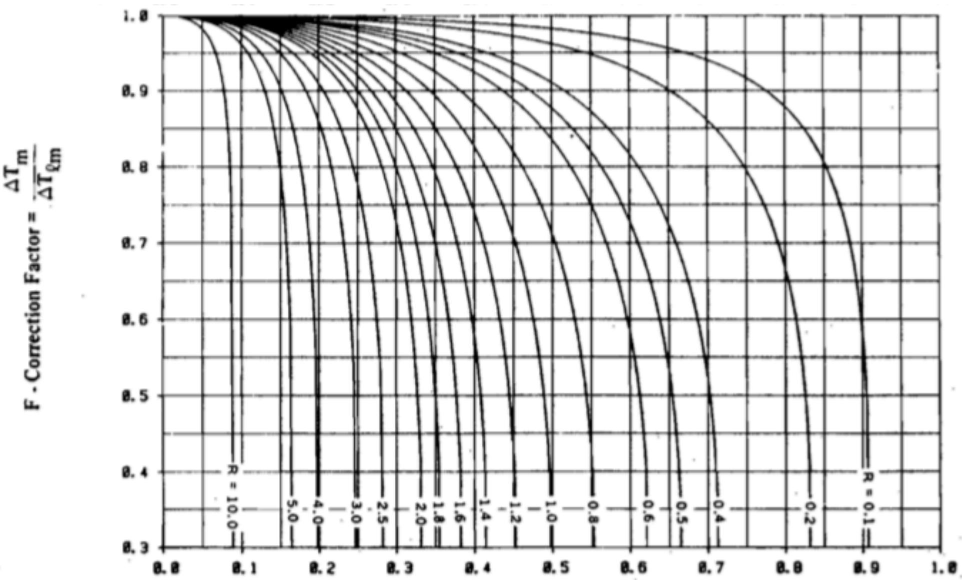

Figure 24 Graphic of co-current flow [16]

Another semi-analytical method is e-NTU (The Number of Transfer Units) method [17] which is used when there is insufficient information to calculated the Log Mean Temperature Difference.

E means efficiency or effectiveness of the heat exchanger, defined as:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} \quad (0 < \varepsilon < 1)$$

The maximum possible heat transfer rate $\dot{Q}_{max}$ is achieved by an ideal heat exchanger with counter flow and infinite heat transfer area. Hence,

$$\dot{Q}_{max} = \left(\dot{m}c_p\right)_{min}(T_{1i} - T_{2i})$$

Similarly, ε in e-NTU method is like F in F-factor method, it depends on flux configuration as well with two other different parameters: NTU (The Number of transfer Units) and heat capacity rate ratio Z.

$$NTU = \frac{U_o A_o}{C_{min}} = \frac{U_i A_i}{C_{min}}$$

$$Z = \frac{C_{min}}{C_{max}}$$

For co-current flow:

$$\varepsilon = \frac{1 - e^{-NTU(1+Z)}}{1 + Z}$$

For counter flow:

$$\varepsilon = \frac{1 - e^{-NTU(1-Z)}}{1 - Ze^{-NTU(1-Z)}}$$

ε can be also be obtained graphically like F. As an example, Figure 25 shows mean temperature difference relationships in a current flow heat exchanger

For any configuration, we have following equation:

$$\dot{Q} = \dot{m}_1 \dot{Cp_1}(T_{1\iota} - T_{Io})$$

$$\dot{Q} = \dot{m}_2 \dot{Cp_2}(T_{2o} - T_{2\iota})$$

$$\dot{Q} = \varepsilon\left(\dot{m}c_p\right)_{min}(T_{1i} - T_{2i})$$

Where,

$$\varepsilon = \varepsilon(configuration, NTU, Z)$$

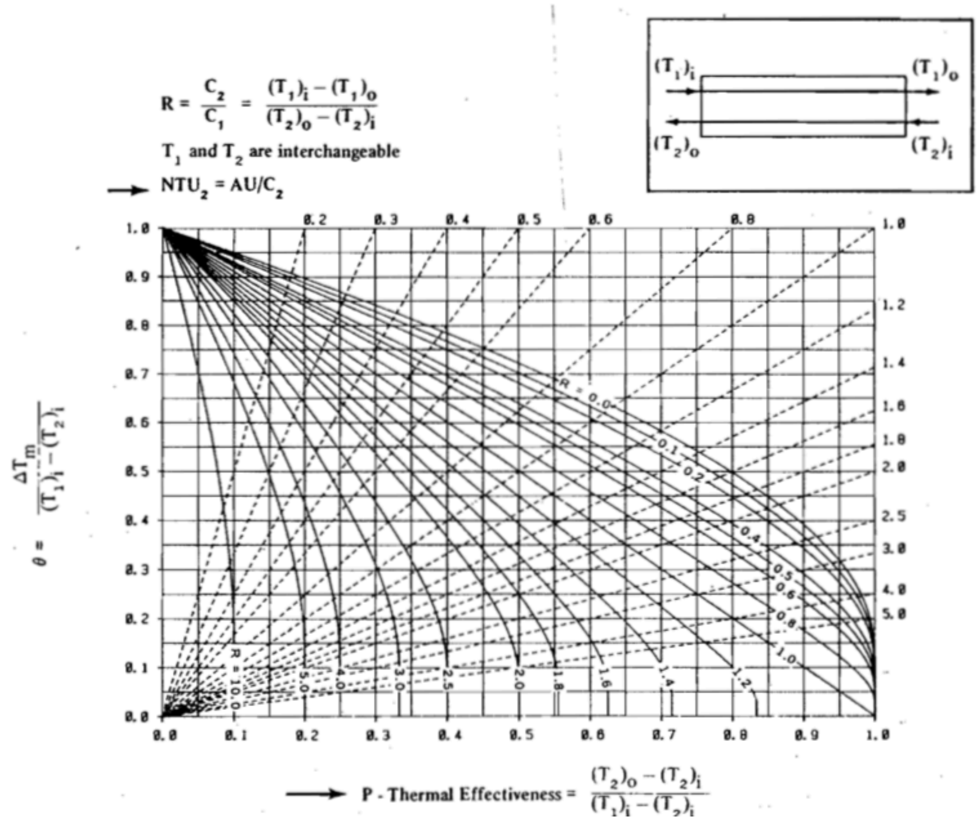

Figure 25 Graphic of counter flow [16]

6.2. Problems of double tube heat exchanger

6.2.1. Problem description

The problem is about a simple co-current flow heat exchanger. Two fluids enter from the entrance with temperature $90\ ℃$ and $15\ ℃$ The objective is to evaluate the outlet temperatures of two fluids and heat transfer power. The geometry condition is listed in the table blow.

Geometry condition	
$D_i(m)$	0.01
$D_o(m)$	0.012
$D_e(m)$	0.02
$l(m)$	5
$v_1(m/s)$	1
$v_2(m/s)$	2
$Uo(W/m^2K)$	800

6.2.2. Numerical method

Similarly, the problem will be solved in both ways: analytical and numerical method. The process of analytical method is simple and this paragraph mainly focus on numerical way. In the numerical method, finite volume is used to divide the domain in finite numbers of control volumes which depends on required precision. The method locates a node at the center of each volume and assumes that the properties of fluids are constant. For co-current flow heat exchanger, discretization equation at each control volume is given below:

$$Fluid\ 1{:}\ Q_i = m_1 c_{p1}(T_1[i] - T_1[i+1]) \tag{1}$$

$$Fluid\ 2{:}\ Q_i = m_2 c_{p2}(T_2[i+1] - T_2[i]) \tag{2}$$

$$Enery\ Exchange{:}\ Q_i = U_o(T_{1\_i} - T_{2\_i})\pi D_o \Delta x \tag{3}$$

When equation (1) and (3) are combined, discretization of fluid 1 is obtained. In the same way, the equation of fluid 2 is obtained:

$$\left(C_1 + \frac{U_o \pi D_o \Delta x}{2}\right) T_1[i+1] = \left(C_1 - \frac{U_o \pi D_o \Delta x}{2}\right) T_1[i] - U_o \pi D_o \Delta x T_{i\_2}$$

The equation could be written into the following form:

$$a_1 T_1[i+1] = b_1[i] - c_1$$

Where:

$$a_1 = \left(C_1 + \frac{U_o \pi D_o \Delta x}{2}\right)$$

$$b_1 = \left(C_1 - \frac{U_o \pi D_o \Delta x}{2}\right)$$

$$c_1 = U_o \pi D_o \Delta x T_{i\_2}$$

Same for the fluid 2:

$$\left(C_2 - \frac{U_o \pi D_o \Delta x}{2}\right) T_2[i+1] = \left(C_1 + \frac{U_o \pi D_o \Delta x}{2}\right) T_2[i] + U_o \pi D_o \Delta x T_{i\_1}$$

$$a_2T_2[i+1] = b_2T_2[i] + c_2$$

Where:

$$a_2 = \left(C_2 - \frac{U_o\pi D_o\Delta x}{2}\right)$$

$$b_2 = \left(C_1 + \frac{U_o\pi D_o\Delta x}{2}\right)$$

$$c_2 = U_o\pi D_o\Delta x T_{i\_1}$$

Actually, same method could be applied to calculate final results in a counter flow heat exchanger and the rest steps are exactly the same as what we did in case of co-current heat exchanger.

$$Fluid\ 1{:}\ Q_i = m_1c_{p1}(T_1[i] - T_1[i+1]) \quad (1)$$

$$Fluid\ 2{:}\ Q_i = m_2c_{p2}(T_2[i] - T_2[i+1]) \quad (2)$$

$$Enery\ Exchange{:}\ Q_i = Q_i = U_o(T_{1\_i} - T_{2\_i})\pi D_o\Delta x \quad (3)$$

6.2.3. Results

Final results of the co-current heat exchanger.

	Analytical results	Numerical results	Differences
T_{o_1}(°C)	62.92	63.02	0.15%
T_{o_2}(°C)	20.13	20.10	0.15%
Q(W)	8656	8624	0.35%

As we could see in the table, there're not big differences between these results with average number less than 1%. However, the performance of heat exchangers doesn't seem satisfactory. Figure 26 shows the temperature distribution along the length of tube, we can see that the change of temperature isn't considerable.

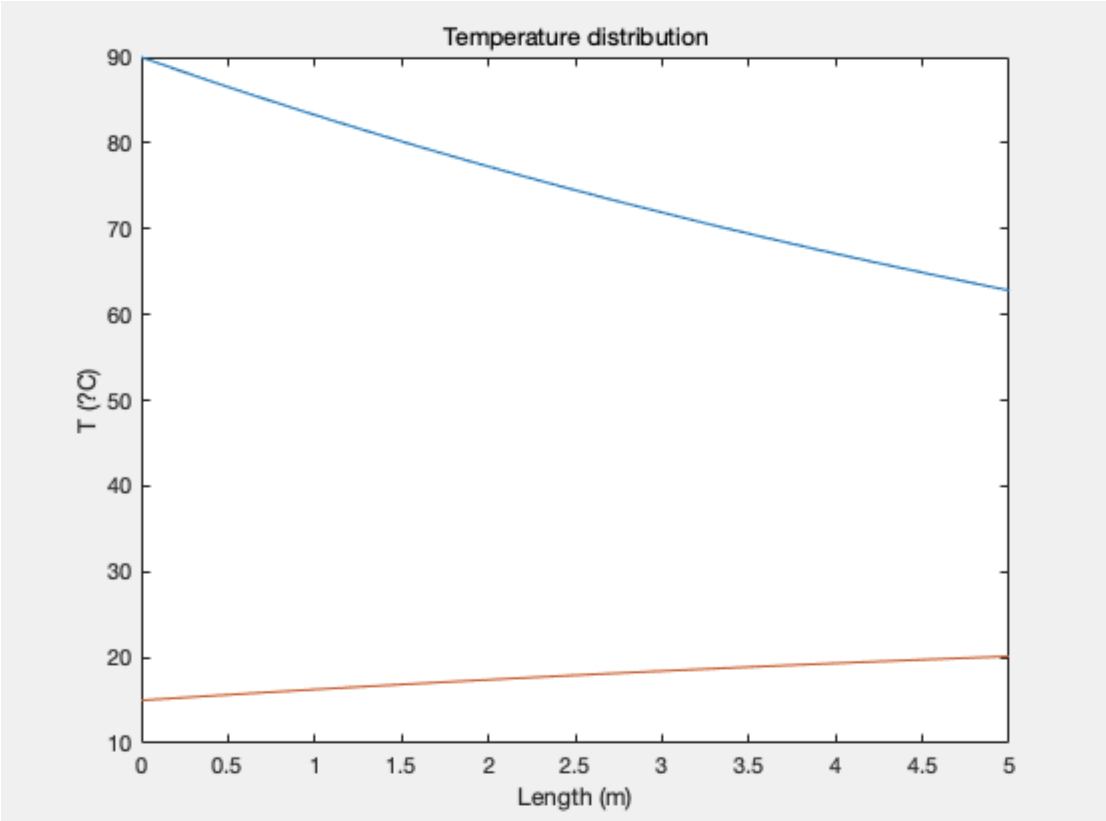

Figure 26 Temperature distribution in co-current flow HX

Using same input data, the outlet temperatures of fluids in a counter heat exchanger are calculated.

	Analytical results	Numerical results	Differences
T_{o_1}(℃)	62.63	62.70	0.11%
T_{o_2}(℃)	20.18	20.03	0.74%
Q(W)	8749	8726	0.26%

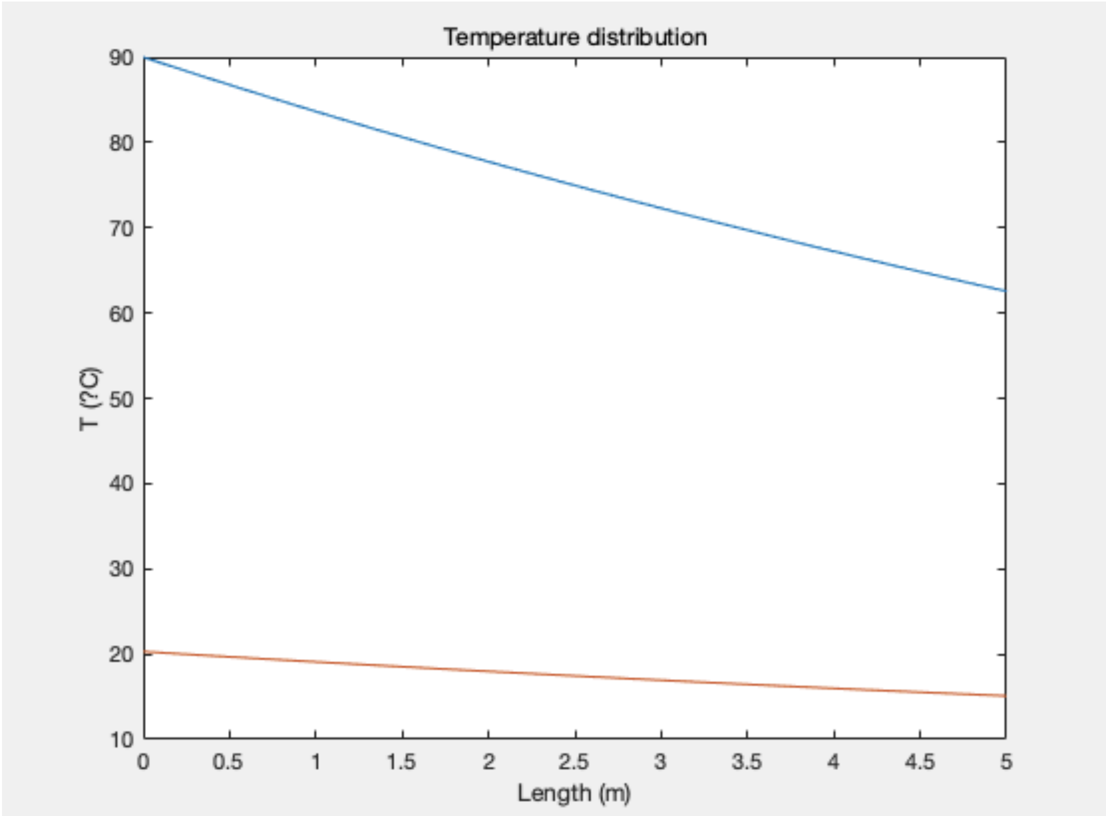

Figure 27 Temperature distribution in counter flow HX

Comparison of two heat exchangers

	Co-current flow	Counter flow	Differences
$T_{o\_1}(℃)$	62.92	62.63	0.46%
$T_{o\_2}(℃)$	20.13	20.18	0.25%
$Q(\mathrm{W})$	8656	8749	1.07%

The table above is the comparison of outlet temperatures in co-current flow and counter flow heat exchangers. We can see that the outlet conditions are close and the heat exchange in counter flow HX is slightly higher than in the co-current flow HX while general speaking, the performance of heat exchanger isn't as considerable as we imagine. One of the biggest problem is the length of heat exchanger which has a crucial influence on the performance. The next part we're going to focus on some other parameters.

6.2.4. Grid sensitivity

The number of nodes in previous basic case is 20 and in this part N=2 and N=200 will be used in numerical calculation to see if N=20 is the most appropriate number of nodes.

Final results of the co-current flow HX with different numbers of nodes

	N=2	N=20	N=200
T_{o_1}(°C)	63.77	63.02	62.98
T_{o_2}(°C)	19.99	20.10	20.11
Q(W)	8796	8624	8672

Difference of co-current flow HX with different numbers of nodes

	N=2	N=20	N=200
T_{o_1}(°C)	1.3%	0.15%	0.09%
T_{o_2}(°C)	0.69%	0.15%	0.10%
Q(W)	1.6%	0.35%	0.18%

From two tables above, it's no difficult to find when 2 nodes are set, the relative differences are larger than basic case and when 200 nodes are divided, the differences are smaller and are close to 0.

Final results of the count flow HX with different numbers of nodes.

	N=2	N=20	N=200
T_{o_1}(°C)	64.01	62.70	62.68
T_{o_2}(°C)	20.53	20.03	20.10
Q(W)	8961	8726	8765

Difference of co-current flow HX with different numbers of nodes

	N=2	N=20	N=200
T_{o_1}(°C)	2.2%	0.11%	0.07%
T_{o_2}(°C)	1.7%	0.74%	0.03%
Q(W)	2.4%	0.26%	0.18%

When increasing number of nodes from 2 to 20, the relative differences become smaller sharping while when it goes 20 to 200, they change slightly.

Like all the previous cases, more nodes made numerical results much closer to analytical results. However, if more nodes are set, more time needed to be invested. Therefore, when it comes to the problem of gird sensitivity, the numbers of nodes will depend on the accuracy required of results.

6.2.5. Parametric study

To study the influence of tube's length on the performance in co-current flow and counter flow heat exchangers, we change the length to 1m and 10m and keep other parameters same using analytical method. The final results came out as following.

Co-current flow

	L=1	L=5	L=10
T_{o_1}(°C)	83.31	62.92	47.47
T_{o_2}(°C)	16.27	20.13	23.05
Q(W)	2139	8656	13594

Counter flow

	L=1	L=5	L=10
T_{o_1}(°C)	83.30	62.63	46.03
T_{o_2}(°C)	16.27	20.18	23.33
Q(W)	2141	8749	14055

From above tables, we could draw conclusion that both kinds of heat exchanger have a similar performance with same inlet conditions and with the increase of tube's length, the performance become better.

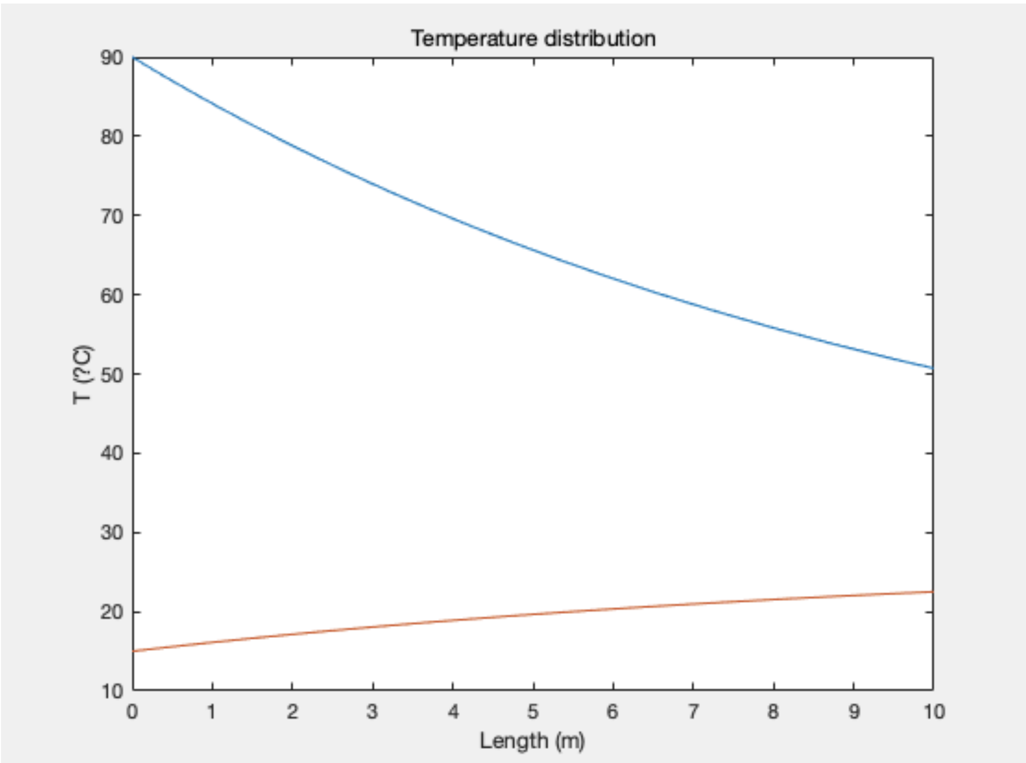

Figure 28 Temperature distribution in co-current flow HX(L=10)

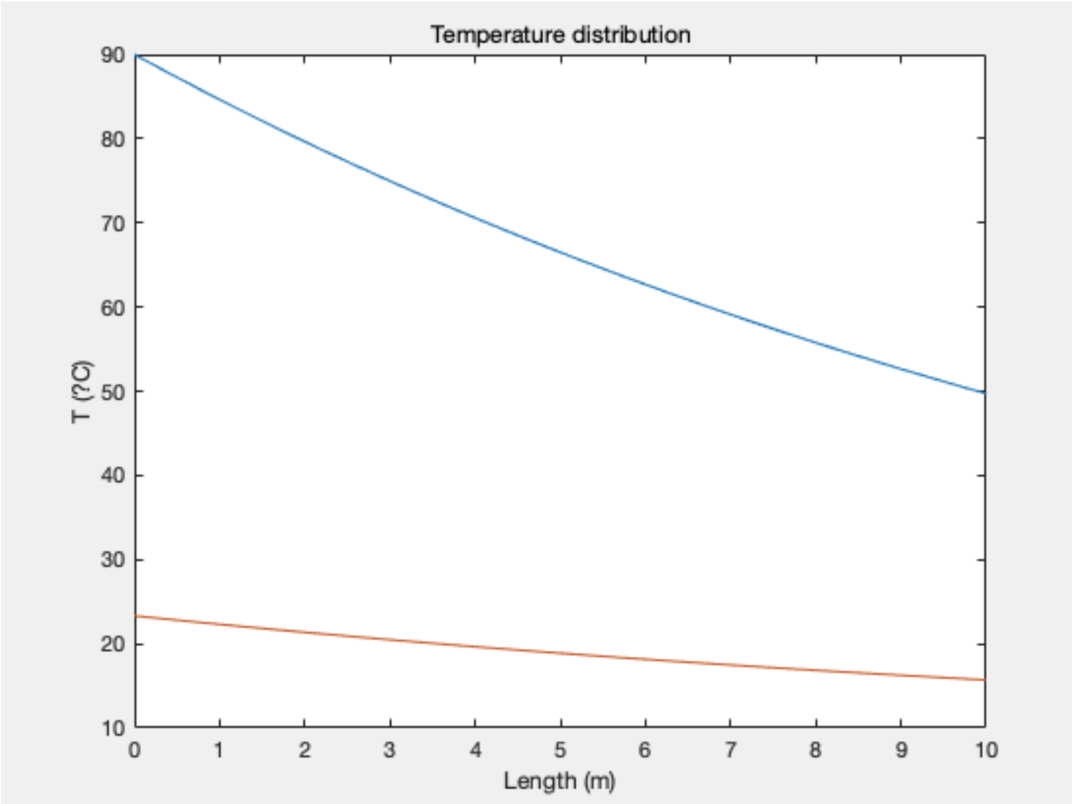

Figure 29 Temperature distribution in counter flow HX(L=10)

Figure 28 and Figure 29 show how the temperature change in the heat exchanger at the length of 10m. It's easy to draw conclusion that the longer the tube is, the more heat will be exchanged between two fluids and if the length of tube approach infinity, the outlet temperatures will be the same and it changes the maximum heat that it could reach.

Diameters of tube

Diameter of tube is another important geometric parameter so in the next part a study of diameters of tube will be presented. New geometry condition is given:

Geometry condition	Basic case	New case
$D_i(m)$	0.01	0.02
$D_o(m)$	0.012	0.024
$D_e(m)$	0.02	0.04

It's easy to find that in the new case the size of tube is twice as the basic case. Using same methods and algorithm, new outlet temperatures are obtained.

Final results of the co-current flow HX with larger size

	Analytical results	Numerical results	Differences
T_{o_1}(°C)	74.57	74.35	0.30%
T_{o_2}(°C)	17.92	17.92	0
Q(W)	19724	20010	1.45%

Comparison of two different sizes

	Basic case	New case	Differences
T_{o_1}(°C)	62.92	74.57	18.52%
T_{o_2}(°C)	20.13	17.92	10.98%
Q(W)	8656	19724	127.87%

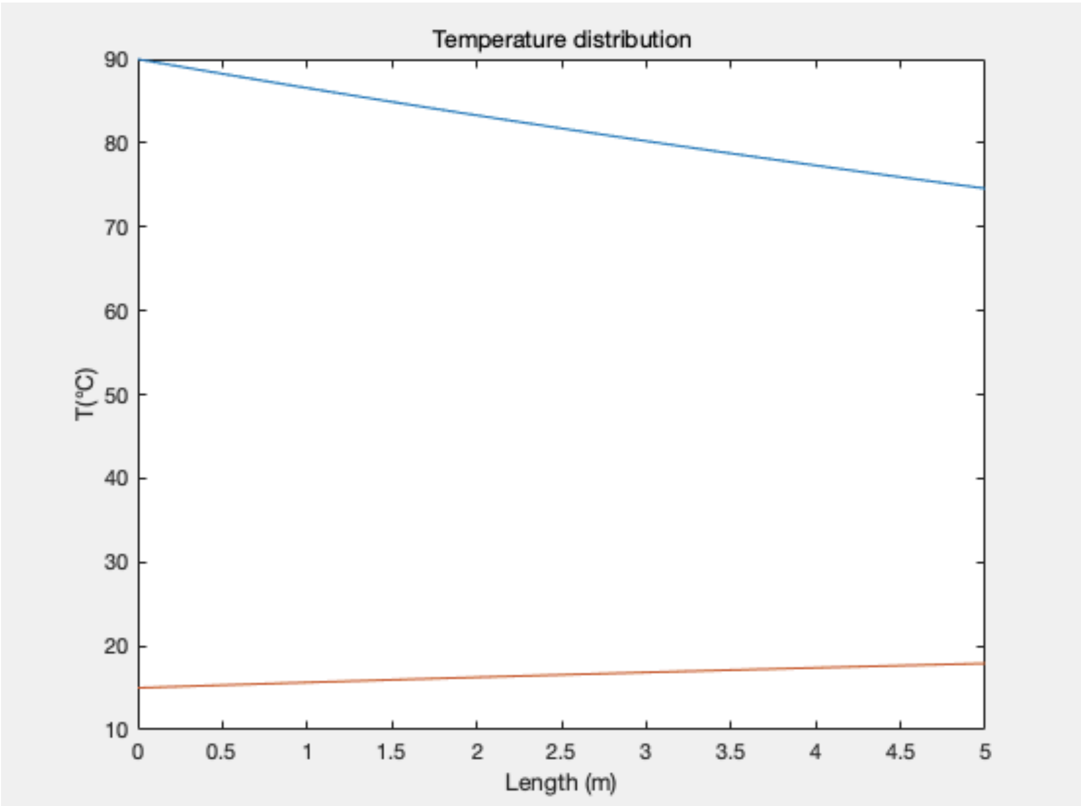

Figure 30 Temperature distribution in co-current flow HX with larger size

From above table and figures, we could find that if the size of tube becomes larger, more heat will be exchanged though the temperature differences are smaller.

Final results of the count flow HX with larger size.

	Analytical results	Numerical results	Differences
T_{o_1}(℃)	74.52	74.42	0.13%
T_{o_2}(℃)	17.93	17.95	0.11%
Q(W)	19784	19920	0.69%

Comparison of two different sizes

	Basic case	New case	Differences
T_{o_1}(℃)	62.63	74.52	18.98%
T_{o_2}(℃)	20.18	17.93	11.14%
Q(W)	8749	19784	126.13%

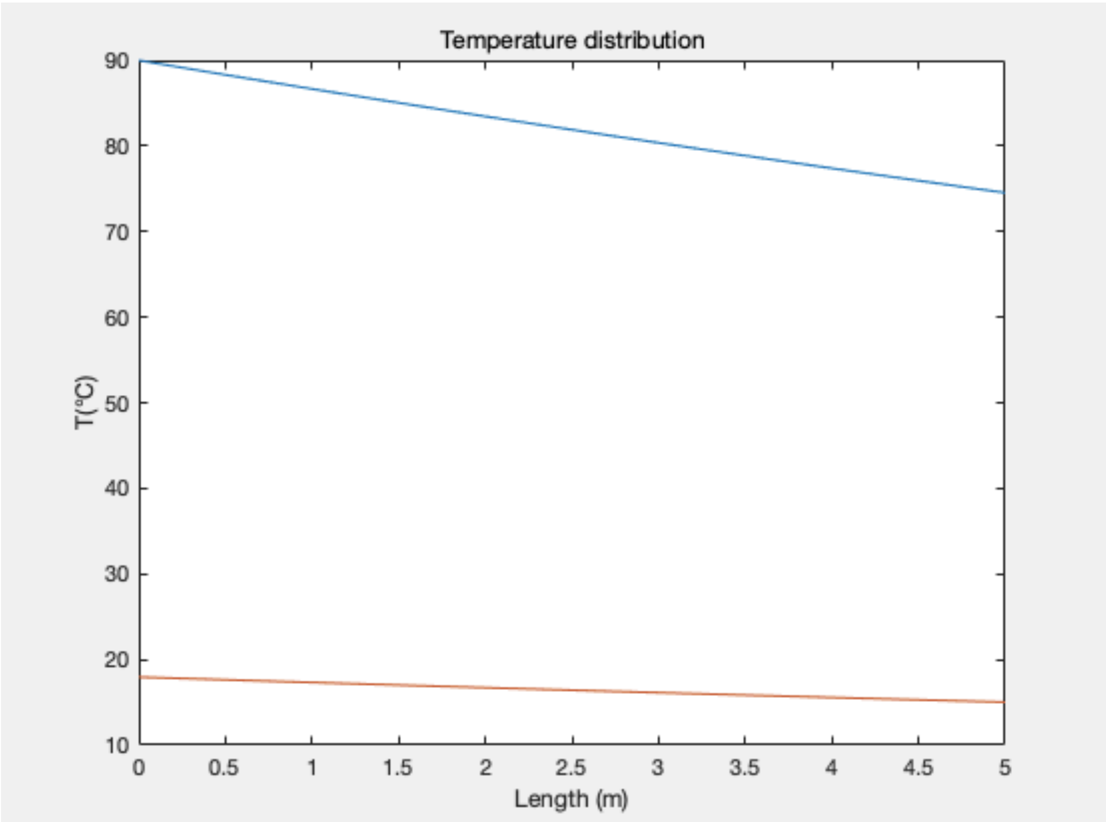

Figure 31 Temperature distribution in counter flow HX with larger size

Same conclusion could be obtained in counter flow that more heat will be transferred while temperature difference is smaller.

Types of fluids

Generally, water is the most used fluid in a double tube heat exchanger for its excellent thermophysical properties and cost-effectiveness while there're also some other fluids that could be applied into a double tube heat exchanger. In the next part, main focus will be on two other fluids: thermal oil 605 and oil 66.

Oil 605

Final results of the co-current flow HX

	Analytical results	Numerical results	Differences
$T_{o\_1}(°\mathrm{C})$	31.25	31.00	0.80%
$T_{o\_2}(°\mathrm{C})$	28.13	28.19	0.21%
$Q(\mathrm{W})$	8167	8202	0.42%3

Final results of the count flow HX

	Analytical results	Numerical results	Differences
$T_{o\_1}(°\mathrm{C})$	42.75	42.78	0.07%
$T_{o\_2}(°\mathrm{C})$	25.56	25.57	0.03%
$Q(\mathrm{W})$	6567	6569	0.03%

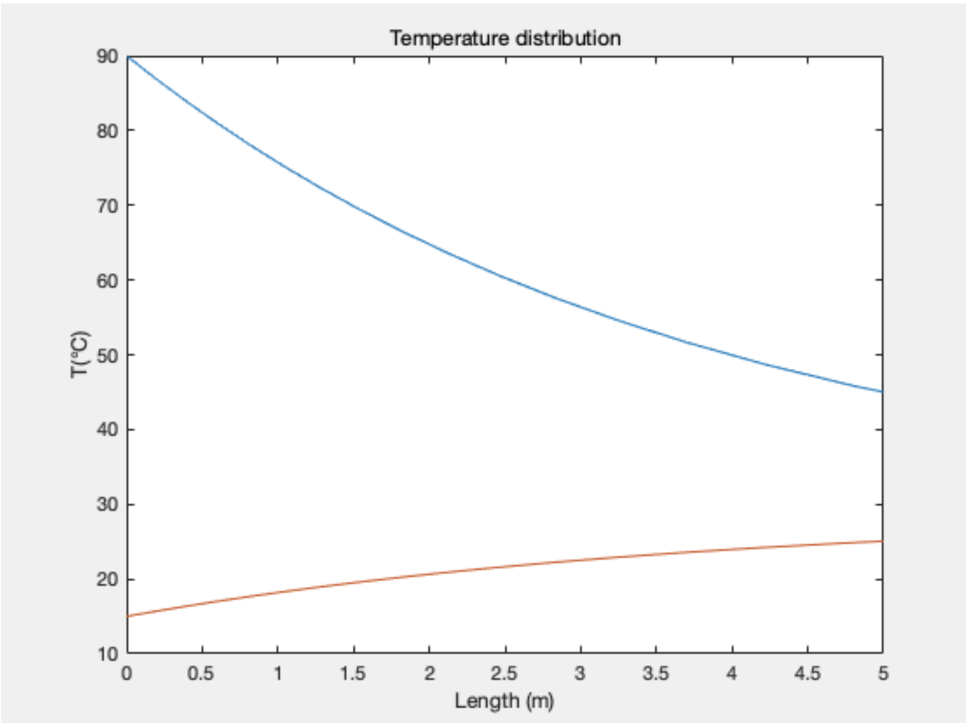

Figure 32 Temperature distribution in counter flow HX using oil 605

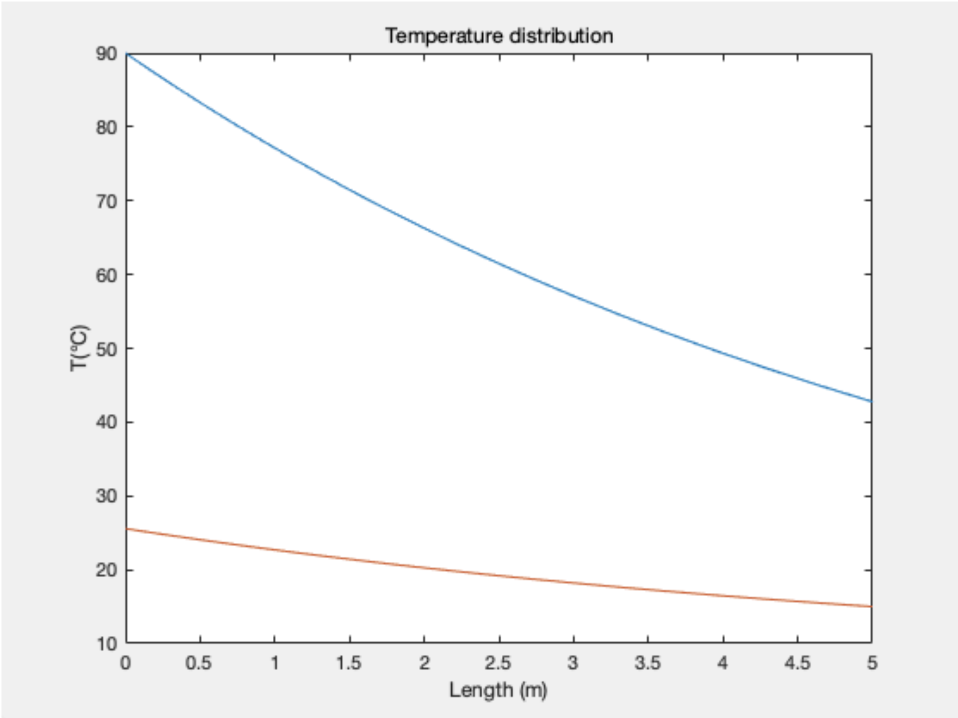

Figure 33 Temperature distribution in counter flow HX using oil 605 Thermal oil 66

Final results of the co-current flow HX

	Analytical results	Numerical results	Differences
$T_{o\_1}(°C)$	31.22	31.05	0.54%
$T_{o\_2}(°C)$	27.74	27.82	0.29%
$Q(W)$	8080	8104	0.30%

Final results of the count flow HX

	Analytical results	Numerical results	Differences
$T_{o\_1}(°C)$	42.39	43.25	2.02%
$T_{o\_2}(°C)$	25.32	25.13	0.75%
$Q(W)$	6545	6427	1.80%

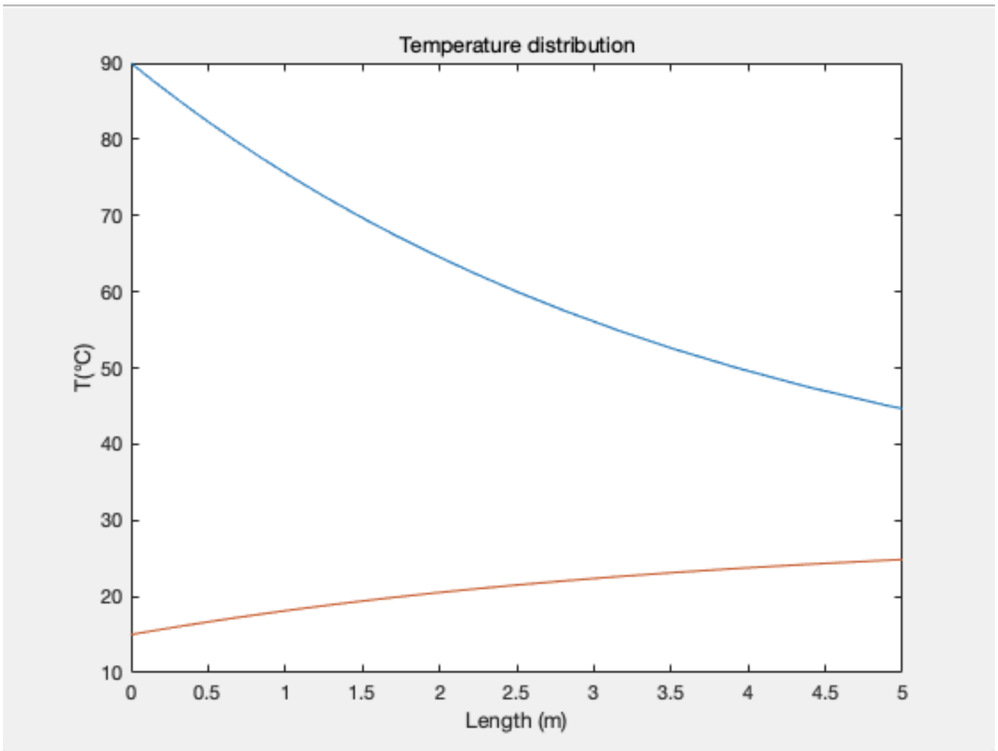

Figure 34 Temperature distribution in counter flow HX using oil 66

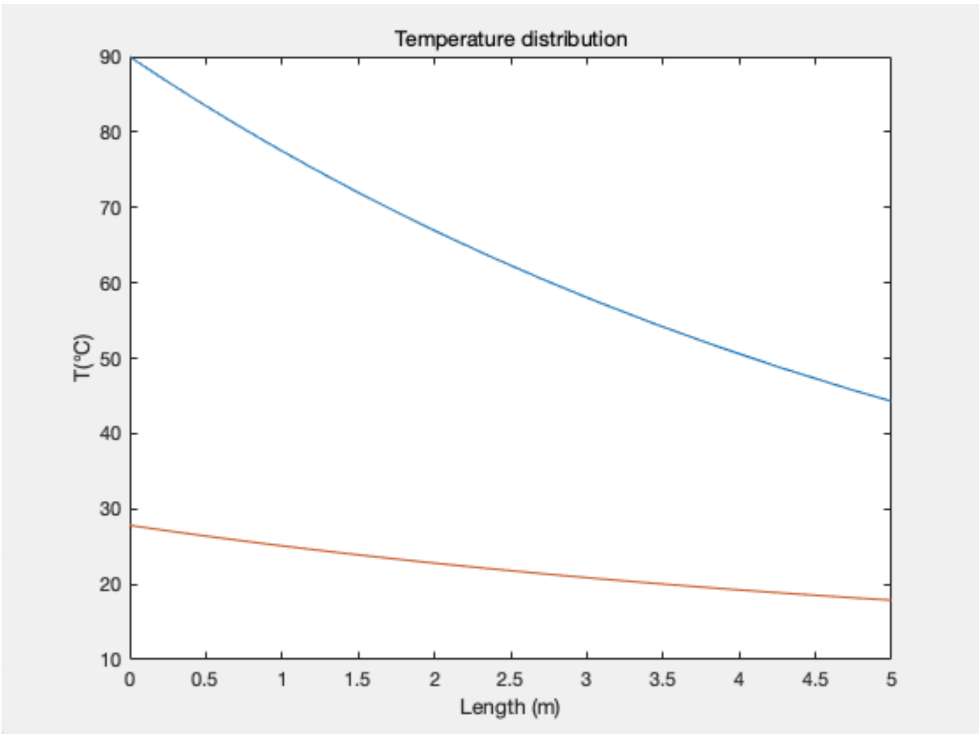

Figure 35 Temperature distribution in counter flow HX using 66

Comparison of the performance using three fluids in co-current flow HX

	Water	Oil 605	Oil 66
$T_{o\_1}(℃)$	62.92	31.25	31.22
$T_{o\_2}(℃)$	20.13	28.13	27.74
$Q(\mathrm{W})$	8656	8167	8080

In co-current flow heat exchanger, it's easy find that using water as fluids has the most heat exchanged while using thermos oils has the better benefit of cooling affect.

Comparison of the performance using three fluids in count flow HX

	Water	Oil 605	Oil 66
$T_{o\_1}(℃)$	62.63	42.75	42.39
$T_{o\_2}(℃)$	20.18	25.56	25.32
$Q(\mathrm{W})$	8749	6567	6545

In counter flow heat exchanger, conclusion is close to the co-current flow but heat exchanged in counter flow is much smaller than in co-current flow using thermos oils.

Conclusions and future actions

Heat conduction

In this chapter, a one-dimensional steady heat conduction problem of composed wall is studied. The temperature distribution along the solid is obtained as well as temperatures at the wall using analytical and numerical method.

The final results from these methods are very close with an average difference less than 1%. What's more, a study about numbers of nodes at the discretization is done and we can find that more numbers of nodes provide more accurate results following by more time costs. As for parametric study, the influence of heat source and thickness of material to the wall temperatures are presented.

Heat convection

In this chapter, a problem of combined forced convection and heat conduction is presented. In the isothermal tube, outlet conditions (temperature, pressure and velocity) are evaluated using step-by-step method and analytical method is also applied to verify the final results. In this problem, three different fluids (water, thermal oil and air) are studied.

According to the results, we can see that with same inlet conditions, the increase of temperature is much larger than the other fluids with a temperature difference more than 70. As for pressure drop, the change in thermal oil is considerable while the others are relatively small. The velocity of water and thermal oil hardly change while it changes sharply in the case of air.

Grid sensitivity is also done in this chapter. According to the results, its easy find that the relative error of outlet conditions decreases with the increase the numbers of nodes and when the number reach 100, the relative error is too small to be neglected. The end of this chapter is a parametric study about length and diameter of tube. The results tell that the performance of longer and thinner tube will be better thank shorter and thicker tube.

Heat exchanger

Last chapter is about double tube heat exchanger. A simple co-current flow and a simple counter flow are studied parallel and work is same as previous chapters: using semi-analytical method and numerical method to calculate outlet temperatures of two fluids in the heat exchanger.

Comparing the results obtained from different methods, we can find that numerical method is reliable that there aren't large differences between these methods. According to data, it shows that the heat exchanged in both HX isn't considerable indicating that the performance is very normal.

Therefore, the parametric study in this problem focus on the length of tube and type of fluids in heat exchangers. The conclusion is that the longer the tube is, the more heat will be exchanged between fluids. If the length of tube is infinite, the maximum heat exchange will happen. As for selection of fluids, water is better.

Future actions

Two-dimension and Three-dimension steady heat conduction could be studied.

Extend to some unsteady cases in heat transfer problems.

Improve skills in writing codes and learn to use CFD or Fluent.

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Appendix

Conduction

```
% Steady one dimensional heat conduction program
% Input data
TgA=40;
TgB=20;
alfaA=10;
alfaB=1000;
e1=0.1;
e2=0.2;
lambda1=0.2;
lambda2=400.0;
qv2=1000;
NA=20;
NB=10;
maxIter=1e6;
maxDiff=1e-10;
dx1=e1/NA;
dx2=e2/NB;

%Mesh
%posX=0:L/(N-1):L;
x=linspace(0,e1+e2,NA+NB+1);

%Coefficients
ap=ones(1,NA+NB+1);
ae=zeros(1,NA+NB+1);
aw=zeros(1,NA+NB+1);
bp=zeros(1,NA+NB+1);
for iX=1:NA+NB+1
    if iX==1
  ae(iX)=lambda1/dx1
  aw(iX)=0
  ap(iX)=ae(iX)+aw(iX)+alfaA
  bp(iX)=alfaA*TgA;
    else if iX==2:NA
  ae(iX)=lambda1/(x(iX+1)-x(iX))
  aw(iX)=lambda1/(x(iX)-x(iX-1))
  ap(iX)=ae(iX)+aw(iX)
  bp(iX)=0;
        else if iX==NA+1
  ae(iX)=lambda2/(x(iX+1)-x(iX))
  aw(iX)=lambda1/(x(iX)-x(iX-1))
  ap(iX)=ae(iX)+aw(iX)
  bp(iX)=0;
            else if iX==NA+2:NA+NB
  ae(iX)=lambda2/(x(iX+1)-x(iX))
  aw(iX)=lambda2/(x(iX)-x(iX-1))
  ap(iX)=ae(iX)+aw(iX)
  bp(iX)=0;
                else iX==NA+NB+1
  ae(iX)=0
  aw(iX)=lambda2/dx2
  ap(iX)=ae(iX)+aw(iX)+alfaB
```

```
  bp(iX)=alfaB*TgB;
              end
          end
      end
  end
end



%Solver
diff=maxDiff+1;
ite=0;
%T=ones(size(x))*((Tw+Te)*0.5);
T=zeros(size(x));
tic;
while diff>maxDiff && ite<maxIter
  Told=T;
  T(1)=(ae(1)*T(2)+bp(1))/ap(1);
  for iX=2:numel(T)-1
    T(iX)=(aw(iX)*T(iX-1)+...
      ae(iX)*T(iX+1)+...
      bp(iX))/ap(iX);
  end
  T(end)=(aw(end)*T(end-1)+bp(end))/ap(end);
%   T=SolverIte(Told,ae,aw,ap,b);
  diff=max(abs(Told-T));
  ite=ite+1;
  if mod(ite,10000)==0
    fprintf('ite: %d diff: %e\n',ite,diff);
  end
end
if ite==maxIter
  warning(['Maximum number of iterations reached
(lastDiff=',num2str(diff),').']);
end
toc;
```

Convection

```
% Convection-Isothermo tube
% case 1 water
% Input data
Di=0.02;
L=5;
V_in=1;
P_in=2.026e5;
T_in=293.15;
Tw=368.15;
N=100;
maxIter=1e6;
maxDiff=1e-10;

% Previous calculation
dx=L/N;
S=pi*(Di^2)/4;
Rou_in=847.2+1.298*T_in-2.657*10^-3*(T_in^2);
Miu_wall=exp(7.867-0.077*Tw+9.04*10^-5*(Tw^2));
```

```
m=Rou_in*S*V_in;

% Mesh
x=linspace(0,L,N);

% Solver
diff=maxDiff+1;
ite=0;
T=zeros(size(x));
P=zeros(size(x));
V=zeros(size(x));
Rou=zeros(size(x));

% Initial map
while diff>maxDiff && ite<maxIter
T(1)=T_in;
P(1)=P_in;
V(1)=V_in;
Rou(1)=Rou_in;
for x=2:numel(T)
    T_old(x)=T(x-1);
    T_ave(x)=0.5*(T(x-1)+T_old(x));
    V_old(x)=V(x-1);
    V_ave(x)=0.5*(V(x-1)+V_old(x));
    P_old(x)=P(x-1);
    P_ave(x)=0.5*(P(x-1)+P_old(x));
    %thermophysical properties
    Rou_ave(x)=847.2+1.298*T_ave(x)-2.657*10^-3*(T_ave(x)^2);
    Cp_ave(x)=5648.8-9.14*T_ave(x)+14.21*10^-3*(T_ave(x)^2);
    Lamda_ave(x)=-0.722+7.168*10^3*T_ave(x)-9.137*10^-6*(T_ave(x)^2);
    Miu_ave(x)=exp(7.867-0.077*T_ave(x)+9.04*10^-5*(T_ave(x)^2));
    % non-dimensionla number
    Re_(x)=Rou_ave(x)*V_ave(x)*Di/Miu_ave(x);
    Pr_(x)=Miu_ave(x)*Miu_ave(x)/Lamda_ave(x);
    Gz_(x)=pi*Di*Re_(x)*Pr_(x)/(4*L);
    if Re_(x)<2000 && Gz_(x)>10
    C=1.86; M=1/3,N=1/3,K=(Di/L)^(1/3)*(Miu_ave(x)/Miu_wall)^0.14;
    elseif Re_(x)<2000 && Gz_(x)<10
         C=3.66, M=0,N=0,K=1;
    elseif Re_(x)>2000 && Pr(x)<=0.6
            C=0.023, M=0.8,N=0.4,K=1;
        else
                C=0.027,M=0.8,N=0.33,K=(Miu_ave(x)/Miu_wall)^0.14;
    end
   Nu_(x)=C*Re_(x)^M*Pr_(x)^N*K;
   alfa_ave(x)=Nu_(x)*Lamda_ave(x)/Di;
   if Re_(x)<2000
       fi(x)=16*Re_(x)^-1;
   else if Re_(x)>5e3 && Re_(x)<3e4
           fi(x)=0.079*Re_(x)^-0.25;
       else if Re_(x)>3e4 && Re_(x)<3e6
               fi(x)=0.046*Re_(x)^-0.2;
           end
   T_new(x)=alfa_ave(x)*(Tw-T_ave(x))*pi*Di*dx/(m*Cp_ave(x))+T_old(x);
   Rou_new(x)=847.2+1.298*T_new(x)-2.657*10^-3*(T_new(x)^2);
   V_new(x)=Rou_old(x)*V_old(x)/Rou_new(x);
   P_new(x)=P_old(x)-
fi(x)*0.5*(Rou_old(x)+Rou_new(x))*(0.5*(V_old(x)+V_new(x)))^2*pi*Di*dx/(2
```

```
*S);
   diff=max(abs(T_new(x)-T_old(x)))
   ite=ite+1;
  if mod(ite,10000)==0
    fprintf('ite: %d diff: %e\n',ite,diff);
  end
end
if ite==maxIter
  warning(['Maximum number of iterations reached
(lastDiff=',num2str(diff),').']);
end
   end
toc;
```

Heat exchangers

```
% Numerical method of cocurrent flow HX
% Input data
Di=0.01;
Do=0.012;
De=0.02;
L=5;
N=20;
Ti_1=90;
Ti_2=15;
V1=1;
V2=2;
maxIter=1e6;
maxDiff=1e-10;
Uo=800; % Global heat coefficient

% Previous calculation
Cp1=5648.79-9.140*(Ti_1+273)+(14.21*10^-3)*(Ti_1+273)^2; % Thermophysical
properites
Cp2=5648.79-9.140*(Ti_2+273)+(14.21*10^-3)*(Ti_2+273)^2;
Rou1=847.2+1.298*(Ti_1+273)-2.657e-3*(Ti_1+273)^2;
Rou2=847.2+1.298*(Ti_2+273)-2.657e-3*(Ti_2+273)^2;
S1=0.25*pi*Di^2;
S2=0.25*pi*(De^2-Do^2);
m1=V1*Rou1*S1;
m2=V2*Rou2*S2;
C1=Cp1*m1;
C2=Cp2*m2;
Ai=pi*Di*L;
Ao=pi*Do*L;

% Semi-analytical method
% Efficiency NTU method
if C1>C2
    NTU=Uo*Ao/C2;
    Z=C2/C1;
else C1<C2
    NTU=Uo*Ao/C1;
    Z=C1/C2;
end
% eff=(1-exp(-NTU*(1+Z))/(1+Z)); % EFFICIENCY COCURRENT FLOW
 eff=(1-exp(-NTU*(1-Z)))/(1-Z*exp(-NTU*(1-Z)));% efficiency counter flow
```

```
if C1>C2
    Q=eff*C2*(Ti_1-Ti_2);
else C1<C2
    Q=eff*C1*(Ti_1-Ti_2);
end
To_1=Ti_1-Q/C1;
To_2=Ti_2+Q/C2;

% Numerical method
dx=L/N; %delta x
x=linspace(0,L,N);
% coefficient of fluid 1
a_1=C1+Uo*pi*Do*dx/2;
b_1=C1-Uo*pi*Do*dx/2;
c_1=-Uo*pi*Do*dx*Ti_2;
% coefficient of fluid 2
a_2=C2+Uo*pi*Do*dx/2;
b_2=C2-Uo*pi*Do*dx/2;
c_2=Uo*pi*Do*dx*Ti_1;
% Solve the problem
diff=maxDiff+1;
ite=0;
% Fluid 1
T=zeros(size(x));
tic;
while diff>maxDiff && ite<maxIter
  Told=T;
  T(1)=Ti_1;
  for iX=2:numel(T)
    T(iX)=(b_1*T(iX-1)+...
     c_1)/a_1;
  end
%   T=SolverIte(Told,a,b,c);
  diff=max(abs(Told-T));
  ite=ite+1;
  if mod(ite,10000)==0
    fprintf('ite: %d diff: %e\n',ite,diff);
  end
end
if ite==maxIter
  warning(['Maximum number of iterations reached
(lastDiff=',num2str(diff),').']);
end

% Fluid 2
x_2=linspace(0,L,N);
T=zeros(size(x));
while diff>maxDiff && ite<maxIter
  Told_2=T;
  T(1)=Ti_2;
  for iX=2:numel(T)
    T(iX)=(b_2*T(iX-1)+...
     c_2)/a_2;
  end
 diff=max(abs(Told_2-T));
  ite=ite+1;
  if mod(ite,10000)==0
    fprintf('ite: %d diff: %e\n',ite,diff);
  end
```

```
end
if ite==maxIter
  warning(['Maximum number of iterations reached
(lastDiff=',num2str(diff),').']);
end
toc;
```